Note Title 2/11/2011

THE PROOF OF THE POWER RULE

* FROM LAST DAY –

1) THE FUNCTION MUST BE CONTINUOUS AT THE POINT WE WISH TO TAKE THE DERIVATIVE OF.

2) THE LIMIT MUST EXIST. *

GIVE $f(x) = ax^n$ PROVE THAT $f'(x) = \boxed{anx^{n-1}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{a(x+h)^n - ax^n}{h}$$

$$= \lim_{h \to 0} \frac{a\left({}_nC_0 x^n h^0 + {}_nC_1 x^{n-1} h^1 + {}_nC_2 x^{n-2} h^2 + \dots {}_nC_{n-1} x^1 h^{n-1} + {}_nC_n x^0 h^n\right) - ax^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{ax^n} + a\,{}_nC_1 x^{n-1} h^1 + a\,{}_nC_2 x^{n-2} h^2 + \dots + a\,{}_nC_n h^n - \cancel{ax^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(a\,{}_nC_1 x^{n-1} + a\,{}_nC_2 x^{n-2} h + \dots + a\,{}_nC_n h^{n-1}\right)}{\cancel{h}}$$

$$= \lim_{h \to 0} a\,{}_nC_1 x^{n-1} + \cancel{a\,{}_nC_2 x^{n-2} h} + \dots \cancel{+ a\,{}_nC_n h^{n-1}})$$

$$= a\,{}_nC_1 x^{n-1}$$

BUT ${}_nC_1 = n$

$= anx^{n-1}$

IE $f(x) = 6x^9$ $\qquad$ $f(x) = \frac{2}{3}x^7$

$f'(x) = 6 \cdot 9 x^{9-1}$ $\qquad$ $f'(x) = \frac{2}{3} \cdot 7 x^6$

$= 54x^8$ $\qquad$ $= \frac{14}{3}x^6$

IE $f(x) = \sqrt[4]{x^6} = x^{\frac{6}{4}} = x^{\frac{3}{2}}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

$f(x) = \frac{1}{x^2} = x^{-2}$ $\qquad$ $f'(x) = -2x^{-3}$