THE PRODUCT & QUOTIENT RULES

Note Title 2/27/2014

GIVEN $f(x) = p(x) \cdot q(x)$

PROVE $f'(x) = p'(x) \cdot q(x) + p(x)\, q'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{p(x+h)\, q(x+h) - p(x)\, q(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{p(x+h)\, q(x+h) - p(x+h)\, q(x) + p(x+h)\, q(x) - p(x)\, q(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{p(x+h)\left(g(x+h) - g(x)\right)}{h} + \frac{g(x)\left(p(x+h) - p(x)\right)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{p(x+h)\left(g(x+h) - g(x)\right)}{h} + \lim_{h \to 0} \frac{g(x)\left(p(x+h) - p(x)\right)}{h}$$

$$f'(x) = \lim_{h \to 0} p(x+h) \cdot \boxed{\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}} + \lim_{h \to 0} g(x) \cdot \lim_{h \to 0} \frac{p(x+h) - p(x)}{h}$$

$$f'(x) = p(x+0) \cdot g'(x) + g(x) \cdot p'(x)$$

$$f'(x) = p(x) \cdot g'(x) + g(x)\, p'(x)$$

<u>IE</u> GIVEN $f(x) = \underbrace{(3x^2 + 5x)}_{1^{ST}}\underbrace{(9x^4 + 3x^7)}_{2^{ND}}$

FIND $f'(x)$

PRODUCT RULE = DERIVATIVE OF THE $1^{ST} \times 2^{ND}$ + DERIVATIVE $2^{ND} \times 1^{ST}$

$$f'(x) = (6x+5)(9x^4+3x^7) + (36x^3+21x^6)(3x^2+5x)$$

QUOTIENT RULE

GIVEN $f(x) = \dfrac{p(x)}{q(x)}$

$$f'(x) = \frac{p'(x)\,q(x) - q'(x)\,p(x)}{\left(q(x)\right)^2}$$

$$\frac{\text{DERIVATIVE OF TOP} \times \text{BOTTOM} - \text{DERIVATIVE BOTTOM} \times \text{TOP}}{(\text{BOTTOM})^2}$$

IE $f(x) = \dfrac{(6x^5 - 5x^9)}{(3x^5 - 6x^2)}$ FIND $f'(x)$

SOLN

$$f'(x) = \frac{(30x^4 - 45x^8)(3x^5 - 6x^2) - (15x^4 - 12x)(6x^5 - 5x^9)}{(3x^5 - 6x^2)^2}$$

IE GIVEN A DEMAND FUNCTION $D(p) = \dfrac{2p + 3000}{10p + 11}$

FIND A) $R(p)$ REVENUE = DEMAND × PRICE

B) $R'(p)$ (MARGINAL REVENUE)

SOLN

A) $R(p) = D(p) \times p$

$$= \left(\frac{2p + 3000}{10p + 11}\right) p$$

$$= \frac{2p^2 + 3000p}{10p + 11}$$

B) $$R'(p) = \frac{(4p + 3000)(10p + 11) - (10)(2p^2 + 3000p)}{(10p + 11)^2}$$

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$$R'(p) = \frac{-16p^2 + 44p + 33000}{100p^2 + 220p - 121}$$