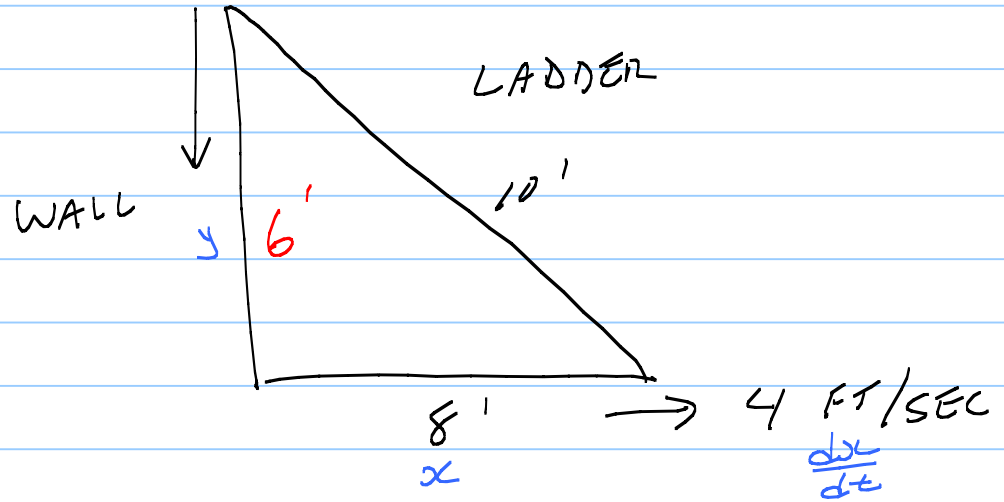
Related Rates

Note Title 4/12/2018

IE A 10 FOOT LADDER, BOTTOM IS MOVING @ 4 FT./SEC. HOW FAST IS THE HEIGHT OF THE TOP DECREASING WHEN THE BOTTOM IS 8 FEET FROM THE WALL?



$$x^2 + y^2 = 10^2$$

$$8^2 + y^2 = 100$$

$$64 + y^2 = 100$$

$$y^2 = 36$$

$$y = 6$$

$$x^2 + y^2 = 10^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(8)(4) + 2(6)\frac{dy}{dt} = 0$$

$$64 + 12\frac{dy}{dt} = 0$$

$$12\frac{dy}{dt} = -64$$

$$\frac{dy}{dt} = \frac{-64}{12}$$

$$\frac{dy}{dt} = -\frac{16}{3} \text{ FT/SEC}$$

- VOLUME OF A BALLOON — WHAT IS THE RATE OF INCREASE IN THE VOLUME @ RADIUS = 3 cm AND RADIUS INCREASING AT A RATE OF 2 cm/s

SOLN

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3 r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi (3)^2 (2)$$

$$\frac{dV}{dt} = 72\pi \text{ cm}^3/\text{s}$$

IE CIRCULAR DISC, RADIUS INCREASING @ 3 cm/sec

WHAT RATE IS THE AREA INCREASING WHEN $r = 4$ cm?

SOLN $A = \pi r^2$

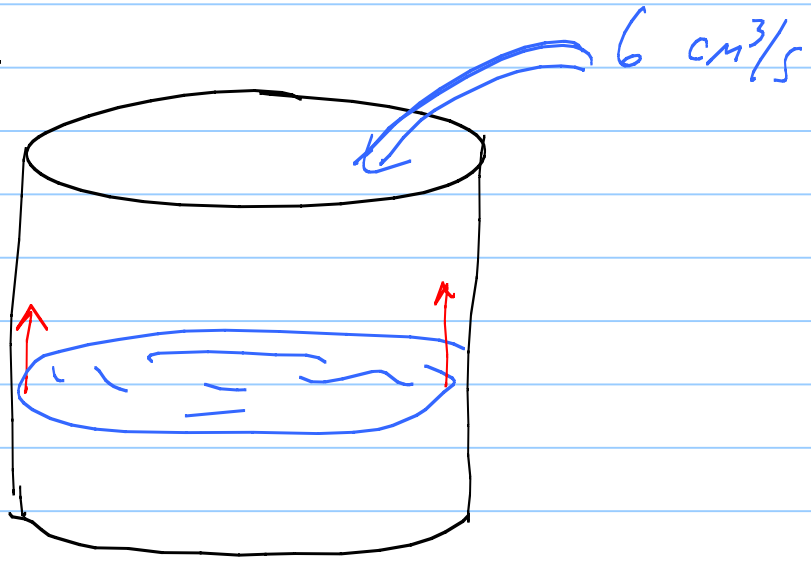
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(4)(3)$$

$$\frac{dA}{dt} = 24\pi \text{ cm}^2/\text{s}$$

IE 4 cm HIGH CYLINDER, 2 cm RADIUS. WATER ADDED @ 6 cm^3/s. AT WHAT RATE DOES THE SURFACE OF THE WATER RISE?

SOLN



$$V = \pi r^2 h$$

(r DOESN'T CHANGE)

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

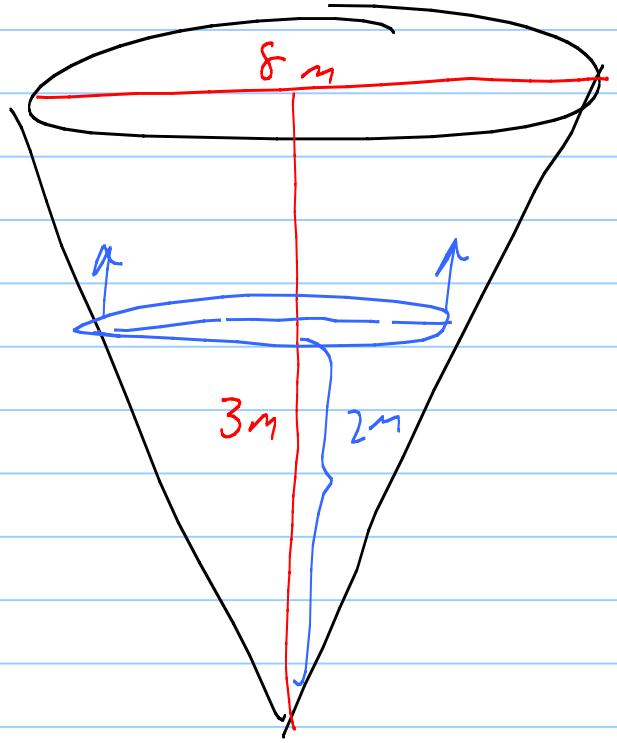
ISOLATE

$$\frac{\frac{dV}{dt}}{\pi r^2} = \frac{dh}{dt}$$

$$\frac{6}{\pi (2)^2} = \frac{dh}{dt}$$

$$\frac{3}{2\pi} \text{ cm/s} = \frac{dh}{dt}$$

IE



FILLING @ 6 m^3/MIN

3 METERS DEEP

8 METER DIAMETER (AT THE TOP)

FIND THE RATE AT WHICH THE SURFACE LEVEL OF THE WATER IS RISING AT THE

INSTANT THE DEPTH IS 2 m

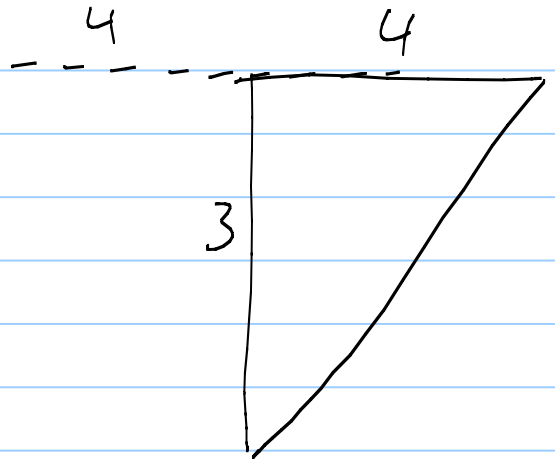
$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 6\ m^3/min$

$h = 2$

$\frac{dh}{dt} = ?$

$r = \frac{4}{3}h$



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{4}{3}h\right)^2 h$

$V = \frac{1}{3}\pi \left(\frac{16}{9}h^2\right) h$

$V = \frac{1}{3}\pi \frac{16}{9} h^3$

$V = \frac{16}{27}\pi h^3$

$$\frac{dv}{dt} = \frac{16}{27}\pi\, 3h^2 \frac{dh}{dt}$$

$$\frac{dv}{dt} = \frac{16\pi}{9} h^2 \frac{dh}{dt}$$

$$\frac{dv}{dt} = \frac{16\pi}{9} (2)^2 \frac{dh}{dt}$$

$$\frac{dv}{dt} = \frac{64\pi}{9} \frac{dh}{dt}$$

$$\frac{64\pi}{9} \qquad \frac{64\pi}{9}$$

$$\frac{dv}{dt} \cdot \frac{9}{64\pi} = \frac{dh}{dt}$$

$$6 \cdot \frac{9}{64\pi} = \frac{dh}{dt}$$

$$\frac{54}{64\pi} = \frac{dh}{dt}$$

$$\frac{27}{32\pi} = \frac{dh}{dt}$$

H/W WORKSHEET

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CHAPTER TEST REVIEW.