IMPLICIT DIFFERENTIATION

Note Title 4/7/2017

RECALL: CHAIN RULE

$$y = (3x^2 + 6x)^4 \qquad \frac{dy}{dx} = 4(3x^2 + 6x)^3 (6x + 6)$$

$$y = (f(x))^5 \qquad \frac{dy}{dx} = 5(f(x))^4 \cdot f'(x)$$

NOW LETS SAY r IS A FUNCTION OF x (NOT A CONSTANT)

$$y = x^2 + 5x + r^3 \qquad \frac{dy}{dx} = 2x + 5 + 3r^2 \cdot \frac{dr}{dx}$$

$$y = x^2 \cdot r \qquad \frac{dy}{dx} = 2x \cdot r + \frac{dr}{dx} \cdot x^2$$

WHAT I AM REALLY INTERESTED IN IS WHEN WE KNOW y IS A FUNCTION OF x

$x^2 + y^2 = r^2$

SO FIND $\frac{dy}{dx}$ OF $x^2 + y^2 = 9$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$
$$-2x \qquad\qquad -2x$$

$$\frac{\cancel{2}y}{2y}\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

DE $\frac{dy}{dx}$ OF $x^2 + 2x - 3y^2 + 2y + 8 = 0$

$$2x + 2 - 6y \cdot \frac{dy}{dx} + 2\frac{dy}{dx} = 0$$
$$-2x \quad -2$$

$$-6y\frac{dy}{dx} + 2\frac{dy}{dx} = -2x - 2$$

$$\frac{dy}{dx}(-6y+2) = -2x-2$$

$$\frac{dy}{dx} = \frac{-2x-2}{-6y+2}$$

$$\frac{dy}{dx} = \frac{\cancel{-2}(x+1)}{\cancel{-2}(3y-1)}$$

$$\frac{dy}{dx} = \frac{x+1}{3y-1}$$

<u>IE</u> FIND THE EQN OF THE TANGENT LINE AT THE POINT $(3,-4)$ ON THE CURVE $x^2+y^2=25$

<u>SOLN</u>

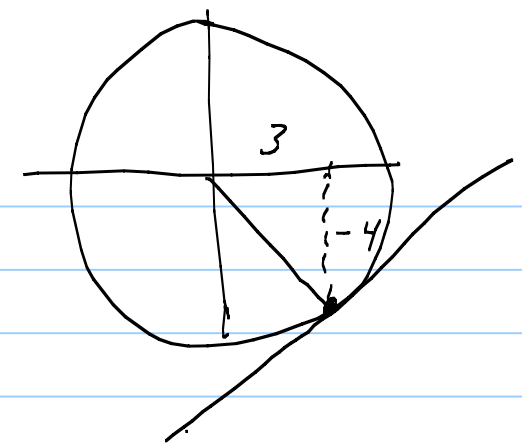
$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$M = \frac{\cancel{-}3}{\cancel{-}4}$$



$$m = \frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - -4 = \frac{3}{4}(x - 3)$$

$$y + 4 = \frac{3}{4}(x - 3)$$

IE FIND $\frac{d^2y}{dx^2}$ OF $x^2 + \underbrace{xy}_{\text{PRODUCT RULE}} + y^2 = 8$

$$(1)(y) + \frac{dy}{dx}(x)$$

$$2x + \overbrace{y + \frac{dy}{dx} \cdot x} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}x + 2y\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx}(x + 2y) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{-2x - y}{x + 2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{\left(-2 - \frac{dy}{dx}\right)(x + 2y) - \left(1 + 2\frac{dy}{dx}\right)(-2x - y)}{(x + 2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left(-2 - \left(\frac{-2x - y}{x + 2y}\right)\right)(x + 2y) - \left(1 + 2\left(\frac{-2x - y}{x + 2y}\right)\right)(-2x - y)}{(x + 2y)^2}$$

H/W p6. 256 #1-18 EVERY THIRD #37