SECTION 5.1 GRAPHING AND SLOPE

A. POINTS AND LINES

In this chapter, we will begin looking at the relationship between two variables. Typically one variable is considered to be the **INPUT**, and the other is called the **OUTPUT**. The input is the value that is considered first, and the output is the value that corresponds to or is matched with the input.

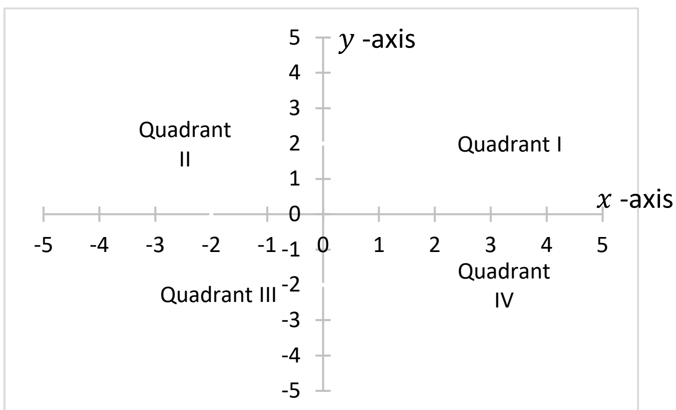
We write the input and its corresponding output as "$(input, output)$." This is known as an ordered pair.

For example,

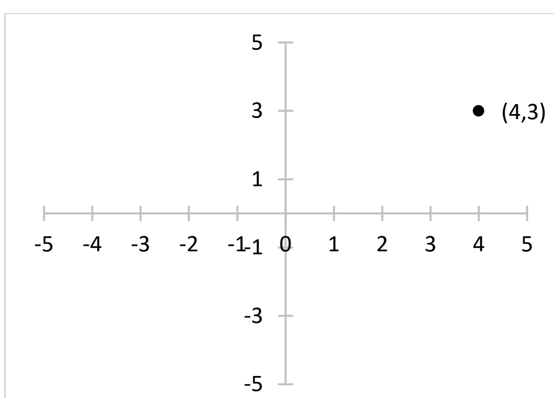
Input	Output	Ordered Pairs
4	-3	$(4,-3)$
5	8	$(5,8)$

In an ordered pair, order matters. Let us take a look at the ordered pair $(4,3)$. Since 4 appears first in this ordered pair, we know that 4 is the input. Likewise, since 3 appears second, we know that 3 is the output that belongs to 4. We can also refer to these numbers as **coordinates**.

To plot ordered pairs we use the **Cartesian plane**. The Cartesian plane is made up of a horizontal real number line (which we call the x-axis) and a vertical real number line (which we call the y-axis). The vertical and horizontal lines intersect at the point $(0,0)$, which is called the **origin**. The Cartesian plane is divided into four **quadrants.**



To plot the ordered pair $(4,3)$ we will look at the first coordinate, 4. We start at the origin and move to the right (the positive direction) by four units. Looking at the second coordinate, 3, we will then go up (in the positive direction) by three units. This is the **point** $(4,3)$.



A **line** is made up of an infinite number of points. To draw a line, however, we only need two points. What a line represents are the solutions to a **linear equation**. An example of a linear equation is

$$y = 2x + 1$$

where x is the input, and y is the output. If we want to graph a linear equation, then we will need to make a table of inputs and outputs. Let us graph the linear equation above. For the table we are creating, we are allowed to pick any inputs we want. One person can pick the input 1 and another can pick the input 1,000. There is no wrong input you can choose for a linear equation, but we would like to keep things as simple as possible. Let us choose the following.

Input (x value)	Output (y value)
0	?
1	?
-2	?

To find the corresponding outputs to the inputs we have chosen, we plug in one x value into the linear equation and solve for y. Let us find all the outputs:

For $x = 0$: $\quad y = 2(0) + 1$
$y = 1$

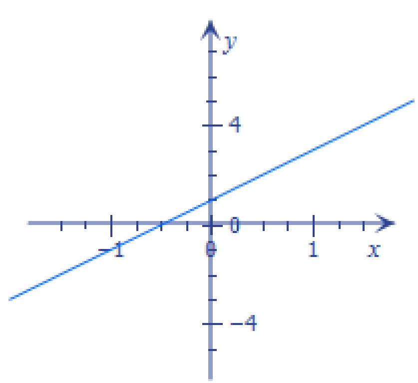
For $x = 1$: $\quad y = 2(1) + 1$
$y = 2 + 1$
$y = 3$

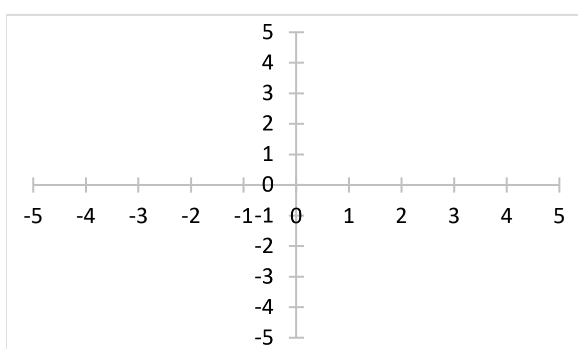
For $x = -2$: $\quad y = 2(-2) + 1$
$y = -4 + 1$
$y = -3$

Filling in our chart

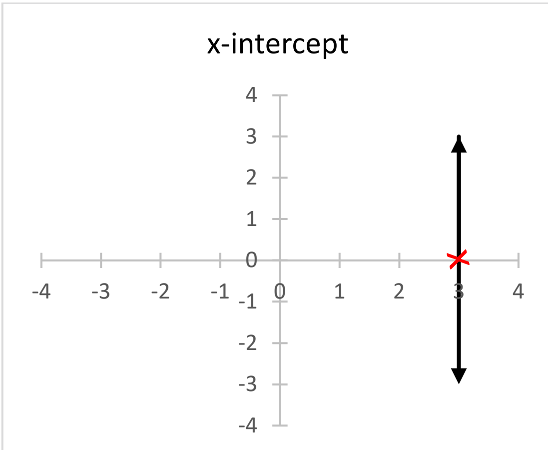
Input (x value)	Output (y value)
0	1
1	3
-2	-3

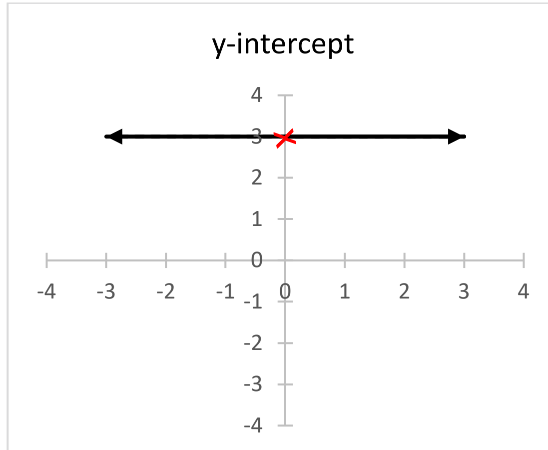
Plotting these ordered pairs allows us to draw the line for the linear equation $y = 2x + 1$





Two important <u>points</u> worth mentioning are the x and y intercepts of the line. The $\boldsymbol{x}$**-intercept** of a line is the <u>point</u> $(x, 0)$, that is, the point where the line crosses the x-axis. The $\boldsymbol{y}$**-intercept** of a line is the <u>point</u> $(0, y)$, that is, the point where the line crosses the y-axis. Below are some examples of x and y intercepts. The cross is indicated by an "x".





stop at 2:57	MEDIA LESSON [Points and lines](#) (Duration 2:57)

View the video lesson, take notes and complete the problems below

- The positive numbers on the x-axis are located in what direction? ______________________

- The negative numbers on the x-axis are located in what direction? ______________________

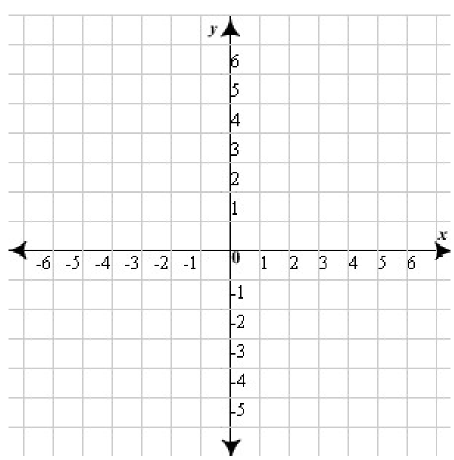
- The positive numbers on the y-axis are located in what direction? ______________________

- The negative numbers on the y-axis are located in what direction? ______________________

We give ________________ to points on the xy-plane using these two number lines. First we give

direction to the point going to ________________, then we give direction to the point going up.

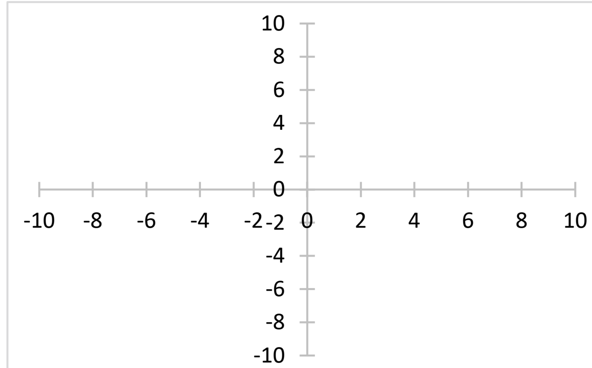
Example: Graph the points. $(-2, 3), (4, -1), (-2, -4), (0, 3)$ and $(-1,0)$



YOU TRY

Plot and label the points.

a) $(-4, 2)$
b) $(3, 8)$
c) $(0, -5)$
d) $(-6, -4)$
e) $(5, 0)$
f) $(2, -8)$
g) $(0, 0)$



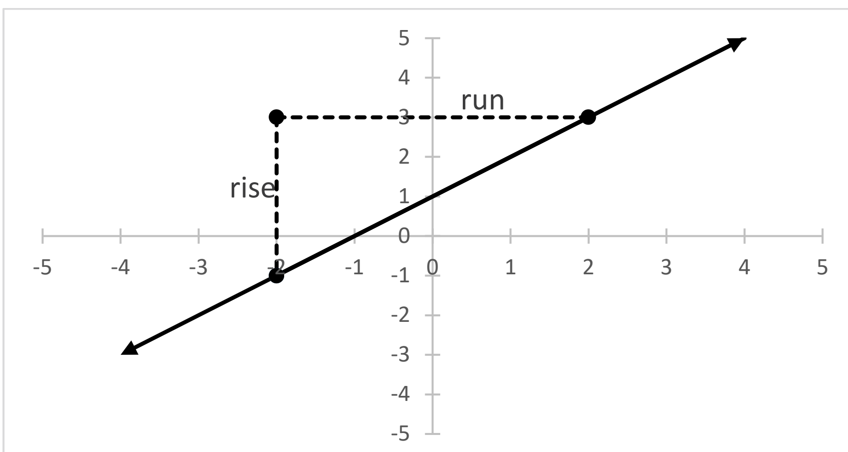
B. OBTAINING THE SLOPE OF A LINE FROM ITS GRAPH

The **slope** of a line is the measure of the line's steepness. We denote the slope of a line with the symbol m. To find the slope of a line from its graph we look at the change in y over the change in x, that is,

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run}$$

In order to determine the rise and run of a graph, let us look at an example. Let us graph the linear equation

$$y = x + 1$$



To find the rise we start at a well-defined point. In our graph above we started at $(-2, -1)$. Then locate a second well-defined point, in our case above we let that second point be $(2, 3)$. Now, starting at our initial point we rise up four units until we get to the exact same level as the second point. This is shown as a dotted vertical line above. Next, we move towards the second point which is four units to the right. This is shown as a dotted horizontal line above.

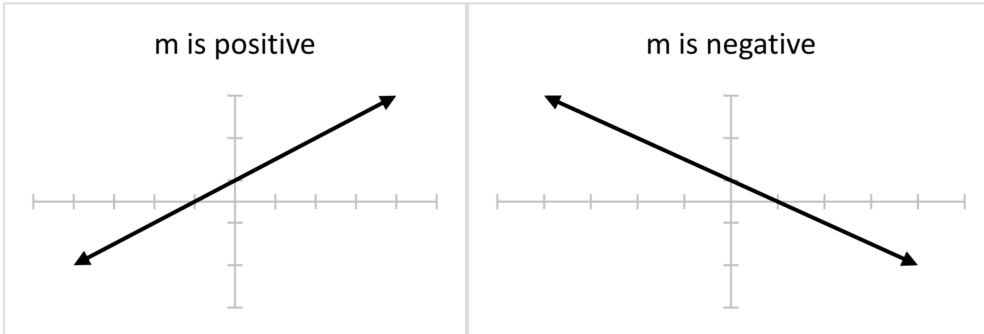
Since we rose up by four units, we say that the rise is 4.

Since we "ran" to the right by four units, we say that the run is 4.
Thus

$$m = \frac{rise}{run} = \frac{4}{4} = 1$$

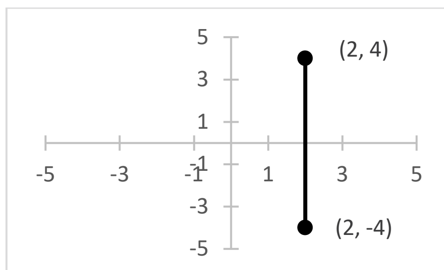
So $m = 1$.

NOTE: If the slope is positive, then the slope will be rising from left to right. If the slope is negative, then the slope will be declining from left to right.



We will now look at two special lines: the vertical line and the horizontal line.

A **vertical line** has the form $x = c$, where c is a constant number. Here is an example of the vertical line $x = 2$



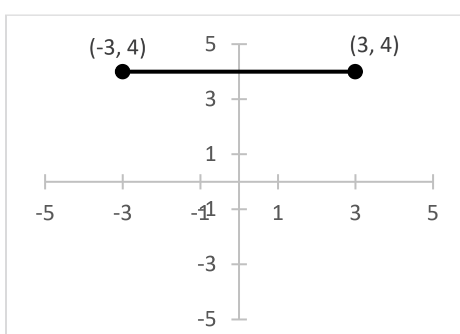
If we were to pick the two well-defined points to be $(2, 4)$ and $(2, -4)$, then the rise would have a value of 8. However, the run will have a value of 0 since we do not move to the right or left.

Thus

$$m = \frac{rise}{run} = \frac{8}{0} = does\ not\ exist$$

Since we can't divide by 0, the slope of the line does not exist.

A **horizontal line** has the form $y = c$, where c is a constant number. Here is an example of the horizontal line $y = 2$.



If we were to pick the two well-defined points to be $(-3, 4)$ and $(3, 4)$, then the rise would have a value of 0 since we do not move up or down. The run, however, will have a value of 6.

Thus

$$m = \frac{rise}{run} = \frac{0}{6} = 0$$

Since 0 divided by anything is 0, our slope does exist and is 0.

To summarize:

- The slope of a vertical line does not exist
- The slope of a horizontal line does exist and has a value of 0.

	MEDIA LESSON Slope from two points (Duration 5:00)

View the video lesson, take notes and complete the problems below

If we select _______________ points on a line we should be able to determine the _______________.

For example, if we are given the coordinates $(3, 3)$ and $(6, 5)$, we should be able to determine the

___________________________________.

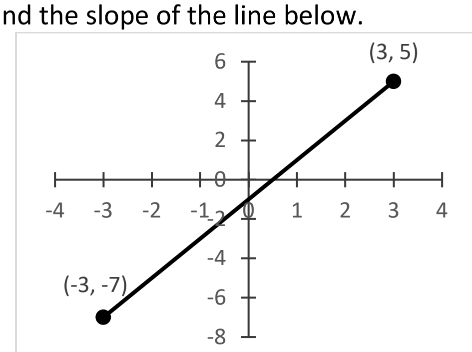
The slope of the two given coordinates is _________________, therefore the y-intercept is equal

to_______________.

We these two pieces of information, the linear equation is _____________________.

YOU TRY

a) Find the slope of the line below.



b) Find the slope of the line below.