DIFFERENTIATION USING LIMITS

Note Title 2/16/2009

REVIEW / NEW STUFF

FACTORING OF THE FORM:

$$(x^3 - y^3) = (x-y)(x^2 + xy + y^2)$$

$$(x^3 + y^3) = (x+y)(x^2 - xy + y^2)$$

IE $\lim_{x \to 2} \dfrac{x^3 - 8}{(x-2)}$

<u>SOLN</u> $\lim_{x \to 2} \dfrac{x^3 - 2^3}{x - 2}$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x^2 + 2x + 2^2)}{\cancel{(x-2)}}$$

$$\lim_{x \to 2} x^2 + 2x + 4$$

$$= 2^2 + 2(2) + 4$$

$$= 12$$

II $$\lim_{x \to 3/2} \frac{8x^3 - 27}{2x - 3}$$

Soln

$$\lim_{x \to 3/2} \frac{(2x)^3 - 3^3}{2x - 3}$$

$$\lim_{x \to 3/2} \frac{\cancel{(2x-3)}\left((2x)^2 + (2x)(3) + (3)^2\right)}{\cancel{(2x-3)}}$$

$$\lim_{x \to 3/2} 4x^2 + 6x + 9$$

$$= 4\left(\frac{3}{2}\right)^2 + \frac{6}{1}\left(\frac{3}{2}\right) + 9$$

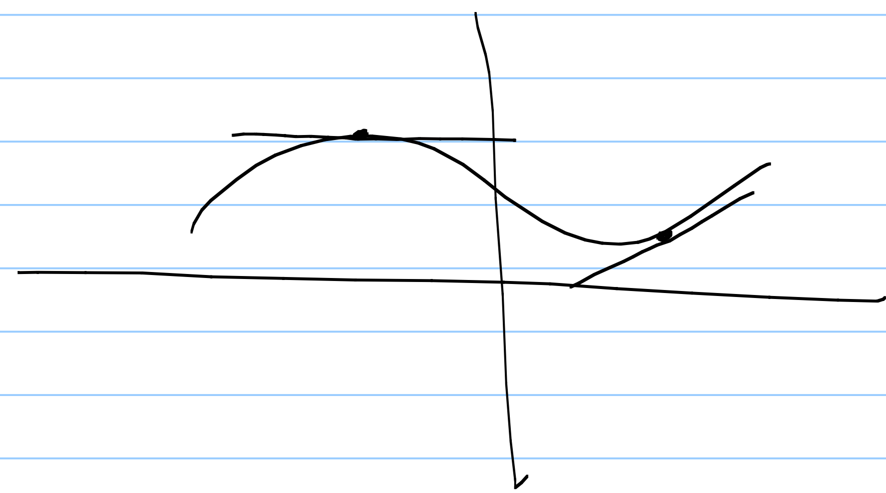
$$= \cancel{4}\left(\frac{9}{\cancel{4}}\right) + \frac{18}{2} + 9$$

$$= 27$$

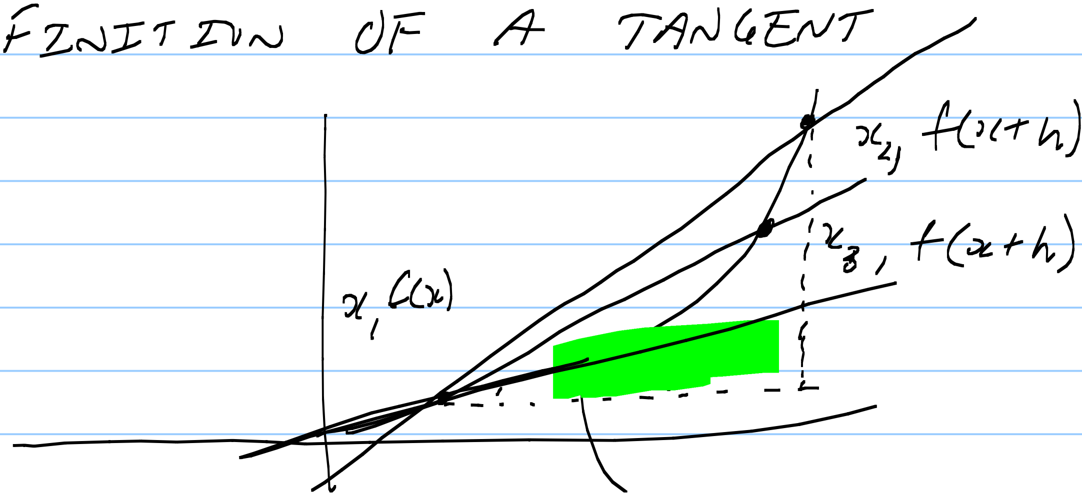
DIFFERENTIATION USING LIMITS

TANGENT LINES – A LINE THAT TOUCHES THE CIRCLE AT EXACTLY ONE POINT.

HOWEVER WE CAN HAVE TANGENT POINTS ON FUNCTIONS



DEFINITION OF A TANGENT



$x_4, f(x+h))$

- THIS IS ALSO THE SLOPE AT THAT POINT.
- THIS IS FORMALLY CALLED THE DERIVATIVE

$\Rightarrow$ THE FUNDEMENTAL THEOREM OF CALCULUS

NOTATION: $f'(x)$ – "f PRIME OF x" $\rightarrow$ THE DERIVATIVE OF y WITH RESPECT TO x

* DERIVATIVE IS THE SLOPE *

YOU WILL ALSO SEE $\frac{dy}{dx}$

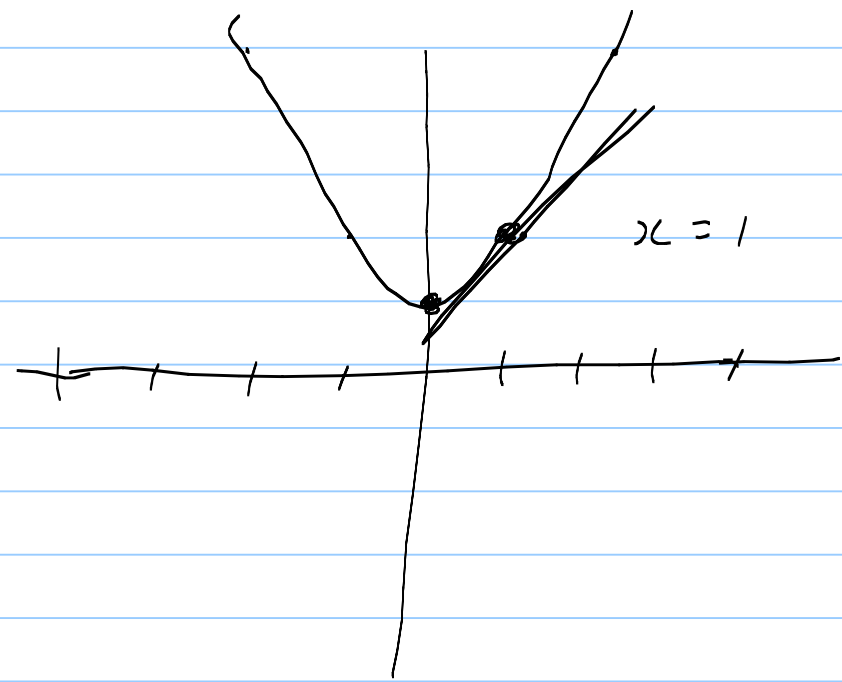
IE GIVEN $y = x^2 + 1$

A) SKETCH THE GRAPH

B) FIND THE SLOPE OF THE TANGENT LINE

SOLN

A)



B)

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+1} \cancel{-x^2} \cancel{-1}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h$$

$$f'(x) = 2x$$

IE $f(x) = 2x^2 + 6x$

A) FIND $f'(x)$

B) FIND TANGENT LINE AT THE POINT $x = 6$

SOLN $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 + 6(x+h) - (2x^2 + 6x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) + 6x + 6h - 2x^2 - 6x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{+6x} + 6h \cancel{-2x^2} \cancel{-6x}}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + 6h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h + 6)}{\cancel{h}}$$

$$= \lim_{h \to 0} 4x + 2h + 6$$

$f'(x) = 4x + 6$ * SLOPE AT ANY POINT *

B) FIND THE $\overline{\text{EQN}}$ OF THE LINE

$(y = mx + b)$

$$(y - y_1) = m(x - x_1)$$

$$m = f'(6) = 4(6) + 6$$

$$f'(6) = 30$$

$$f(x) = 2x^2 + 6x$$

$$f(6) = 2(6)^2 + 6(6)$$

$$f(6) = 108$$

$$(y - 108) = 30(x - 6)$$

H/W Pg 125

#1, 6, 8, 14–16, 18, 23, 24, 30, 32