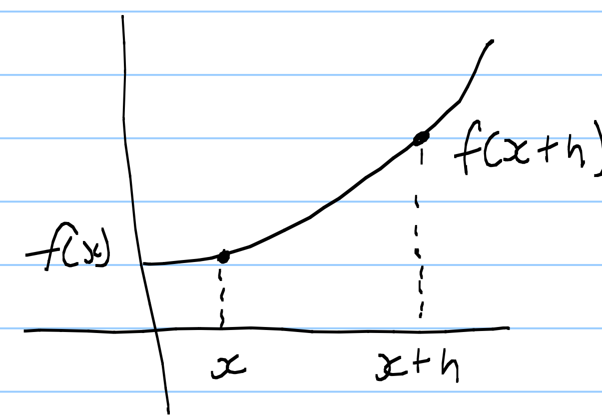
Note Title 4/12/2016

DIFFERENTIALS

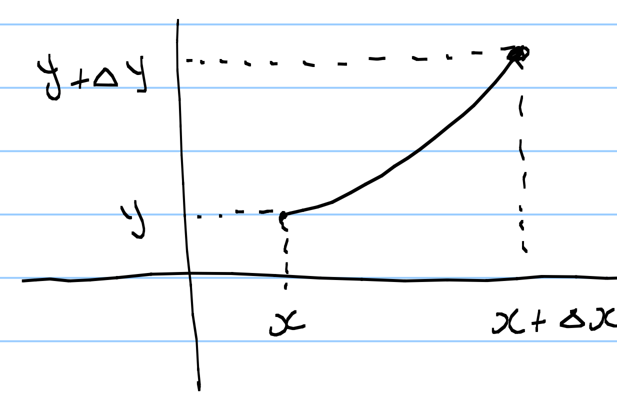
RECALL THE DEF$\bar{n}$ OF THE DERIVATIVE

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



THIS CAN ALSO BE WRITTEN AS

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



OR $\dfrac{\Delta y}{\Delta x}$

SO WE ALSO KNOW THAT $f'(x)$ CAN BE WRITTEN

AS $\dfrac{dy}{dx}$ $\therefore$ FOR SMALL VALUES OF Δx

$$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x} \quad \text{OR} \quad f'(x) \approx \frac{\Delta y}{\Delta x}$$

$$\therefore \quad \Delta y = f'(x)\,\Delta x$$

IE APPROXIMATE $\sqrt{27} = 5 + \underline{\qquad}$

$f(x) = \sqrt{x}$ $\qquad$ $x = 25$

$\Delta x = 2$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$= \dfrac{1}{2x^{\frac{1}{2}}}$

$\Delta y = f'(x)\,\Delta x$

$\Delta y = f'(25) \cdot 2$

$= \frac{1}{2\sqrt{x}}$

$\Delta y = \frac{1}{\cancel{2}\sqrt{25}} \cdot \cancel{2}$

$\Delta y = \frac{1}{5}$

$\therefore \sqrt{27} \approx 5\frac{1}{5}$

II APPROXIMATE $\sqrt{50} = 7 + \_\_\_$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$x = 49$

$\Delta x = 1$

$\Delta y = f'(x)\,\Delta x$

$\Delta y = f'(49) \cdot 1$

$\Delta y = \frac{1}{2\sqrt{49}} \cdot 1$

$\Delta y = \frac{1}{14}$

$\therefore \sqrt{50} \approx 7\frac{1}{14}$

IE $y = x(4-x)^3$ FIND $\frac{dy}{dx}$ AND dy WHEN $x=5$
AND $dx = 0.2$

SOLN

$$\frac{dy}{dx} = (1)(4-x)^3 + 3(4-x)^2(-1)(x)$$

$$\frac{dy}{dx} = (4-x)^3 - 3x(4-x)^2$$

$$\frac{dy}{dx} = (4-x)^2\left[(4-x) - 3x\right]$$

$$\frac{dy}{dx} = (4-x)^2(-4x+4)$$

$$\frac{dy}{dx} = -4(4-x)^2(x-1)$$

$$dy = -4(4-x)^2(x-1)\,dx$$

$$dy = -4(4-(5))^2(5-1)(0.2)$$

$$dy = -3.2$$

H/W Pg 250

15, 16, 19, 20 DUE E.O.C.

21 - 28 DUE WED.