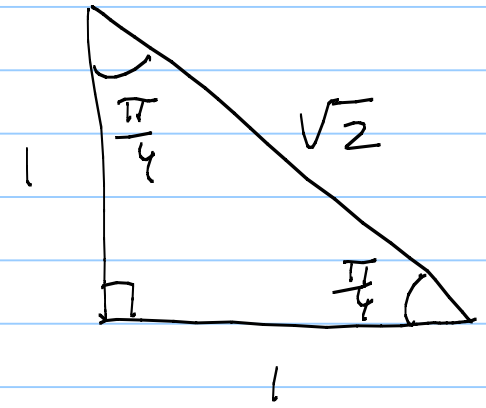
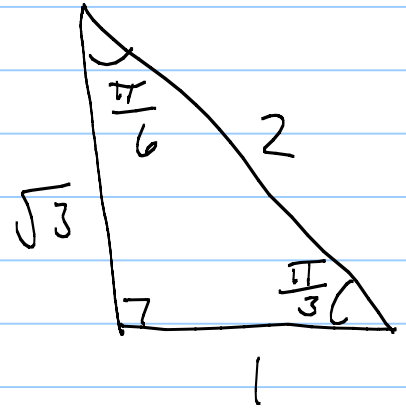
Derivatives of Trig Functions

Note Title 4/23/2019

From Pre-Calc 12





Also $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \csc\theta = \frac{1}{\sin\theta}$

- PLUS ALL THE IDENTITIES ON THE WALL

IE PROVE $\frac{d}{dx}\sin x = \cos x$ USING DEFN OF DERIVATIVE

SOLN IF WE GRAPH $y = \frac{\sin h}{h}$ WE WOULD FIND

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \qquad \text{ALSO} \qquad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

$$\text{so} \quad \frac{d}{dx}\sin x = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

BY USING SUM IDENTITY $\sin(x+y)$ WE GET

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

REARRANGE $= \lim_{h \to 0} \dfrac{\sin x \cos h - \sin x + \cos x \sin h}{h}$

$= \lim_{h \to 0} \dfrac{\sin x \cos h - \sin x}{h} + \dfrac{\cos x \sin h}{h}$

$= \lim_{h \to 0} \dfrac{\sin x (\cos h - 1)}{h} + \dfrac{\cos x \sin h}{h}$

FROM ABOVE $= \sin x \lim_{h \to 0} \dfrac{\cos h - 1}{h} + \cos x \lim_{h \to 0} \dfrac{\sin h}{h}$

$= \sin x (0) + \cos x (1)$

$\dfrac{d}{dx} \sin x = \cos x$

OTHER DERIVATIVES ARE:

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \tan x \sec x \qquad \frac{d}{dx}\csc x = -\cot x \csc x$$

IE PROVE $\frac{d}{dx}\sec x = \tan x \sec x$

SOLN $\frac{d}{dx}\frac{1}{\cos x}$

$$\frac{(0)(\cos x) - (-\sin x)(1)}{(\cos x)^2}$$

$(\cos x)^2 = \cos^2 x$

$$\frac{\sin x}{\cos^2 x}$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$\tan x \sec x$$

PROVE $\frac{d}{dx} \cot x = -\csc^2 x$

SOLN $\frac{d}{dx} \frac{\cos x}{\sin x}$

$$\frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x}$$

$$\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$\frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$\frac{-1}{\sin^2 x}$$

$$-\csc^2 x$$

IE FIND THE DERIVATIVE OF $y = \sin x \cos x$

SOLN $$\frac{dy}{dx} = (\cos x)(\cos x) + (-\sin x)(\sin x)$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x$$

IE FIND $\frac{dy}{dx}$ OF $y = \sec^3 x$

SOLN USE CHAIN RULE

$$\frac{dy}{dx} = 3\sec^2 x \left(\frac{dy}{dx}\sec x\right)$$

$$= 3\sec^2 x \left(\tan x \sec x\right)$$

$$= \left(\frac{3}{\cos^2 x}\right)\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right)$$

$$= \frac{3\sin x}{\cos^4 x}$$

H/W Pg 560

7, 9, 13, 17, 19, 23, 29, 33, 34, 41, 44

1, 2 DUE ~~TOMORROW~~ THURSDAY