	Pre-Calculus 12	
	Trigonometric Equations & Proofs – The really big review…	

6.1 Reciprocal Identities.

1. Simplify: $\dfrac{\sin x \cot x}{\sec x}$
2. Verify: $\dfrac{1}{\sec x \tan x} = \sec x - \sin x$
3. Prove: $\sin x + \cos x \cot x = \csc x$

6.2 Pythagorean Identities.

1. Simplify: $\sin^2 x \sec^2 x - \sin^2 x$
2. Verify: $\sin^2 x - 1 = \cos^2 x$
3. Prove: $\dfrac{1}{\cos x} - \cos x = \dfrac{\tan x}{\csc x}$
4. Prove: $\dfrac{\sec x}{1-\cos x} = \dfrac{\sec x + 1}{\sin^2 x}$

6.3 Sum/Difference Identities.

1. Evaluate & simplify with special triangles and graphs:

a. $\sin\left(\dfrac{\pi}{2} + x\right)$

b. $\tan\left(x + \dfrac{\pi}{3}\right)$

c. $\cos\left(x + \dfrac{\pi}{4}\right)$

3. Given $\cos A = \dfrac{5}{13}$, and A is in quadrant IV, find the value of:

a. $\sin\left(A + \dfrac{\pi}{2}\right)$

b. $\tan\left(A + \dfrac{\pi}{3}\right)$

c. $\csc\left(A + \dfrac{\pi}{2}\right)$

6.4 Double Angle Identities

1. Simplify (or write another way that is more compact)

a. $\dfrac{2\sin x}{\sin 2x}$

b. $\left(\cos^2 x - \cos 2x\right)\csc x$

c. $\dfrac{8\tan(10x)}{4 - 4\tan^2(10x)}$

2. Given $\cos A = \dfrac{5}{13}$, and A is in quadrant IV, find the value of:

a. $\sin(2A)$

b. $\csc(2A)$

c. $\cos(2A)$

d. $\sec(2A)$

e. $\tan(2A)$

f. $\cot(2A)$

3. Prove: $\sin^2 x - \cos^2 x = 2\sin^2 x - 1$

4. Prove: $\dfrac{2\cos x + 2\cos^2 x}{\sin 2x} = \dfrac{\sin x}{1 - \cos x}$

6.5 Restrictions.

1. Find the restrictions in terms of sine and cosine: $\dfrac{\cos x}{1 + \sin x}$
2. Find the restrictions in terms of sine and cosine: $\dfrac{\sec x}{1 - \cos x}$
3. Find the locations of the vertical asymptotes (in terms of x) on the graph of: $y = \cot x + \tan x$

6.6 Solving Llinear Trigonometric Equations.	
1. Solve $3\sin x+2=0$ (accurate to 2 decimal places), over the domain: a. $0\le x<2\pi$ b. $0°\le x<360°$	2. Solve $\tan x=-\sqrt{3}$ (as exact values), over the domain: a. $0\le x<2\pi$ b. $-\pi\le x<\pi$ c. $-\frac{\pi}{2}<x<\frac{\pi}{2}$ d. $0°\le x<360°$
6.7 Solving by Factoring.	
1. Solve $\sqrt{3}\sec x\sin x-2\sin x=0$ (as exact values), over the domain: a. $0\le x<2\pi$ b. $-\pi\le x<\pi$ c. $-\frac{\pi}{2}<x<\frac{\pi}{2}$ d. $0°\le x<360°$	2. Solve $3\cos^2 x-8\cos x-3=0$ (accurate to 2 decimal places), over the domain: a. $0\le x<2\pi$ b. $0°\le x<360°$
6.8 Using Double Angle Identities (how period affects solutions):	
1. Give the general solution to: $2\sin(3x)-\sqrt{3}=0$	2. Give the general solution to: $4\cos(5x)-2=0$
3. Use an identity to give the general solution to: $8\sin x\cos x-4=0$	4. Use an identity to solve: $\sin x=\cos(2x)$ over the domain: $0\le x<2\pi$
6.9 Using technology	
1. Solve (show the **GRAPH**): $\log_2(x)=\cos x$ over the domain: $0\le x<2\pi$	2. Solve (show the **GRAPH**): $\sec x+\tan x=\sin x$ over the domain: $0\le x<2\pi$

KEY

Chapter 6.1

1. Simplify: $\dfrac{\sin x \cot x}{\sec x}$

$$= \frac{\cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}}}{\frac{1}{\cos x}}$$

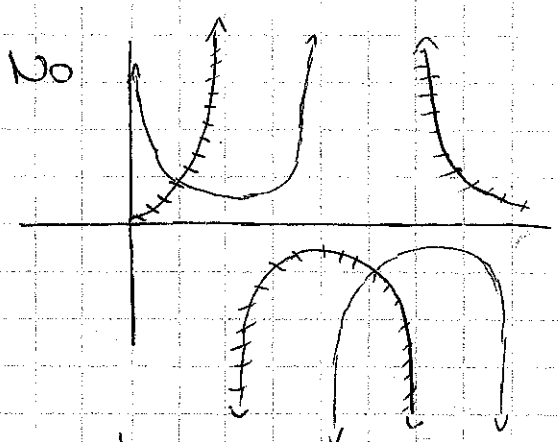
$$= \frac{\cos x}{\frac{1}{\cos x}}$$

$$= \cos x \cdot \frac{\cos x}{1}$$

$$= \cos^2 x$$

2. If $\dfrac{1}{\sec x \tan x} = \sec x - \sin x$ their graphs should be the same.

Ans: No



$= \dfrac{1}{\sec x \tan}$ $= \sec x - \sin x$

3. $\sin x + \cos x \cot x = \csc x$

$= \sin x + \cos x \cdot \dfrac{\cos x}{\sin x}$	$\dfrac{1}{\sin x}$
$= \sin x + \dfrac{\cos^2 x}{\sin x}$	
$= \dfrac{\sin^2 x + \cos^2 x}{\sin x}$	
$= \dfrac{1}{\sin x}$	$= \dfrac{1}{\sin x}$

Chapter 6.2

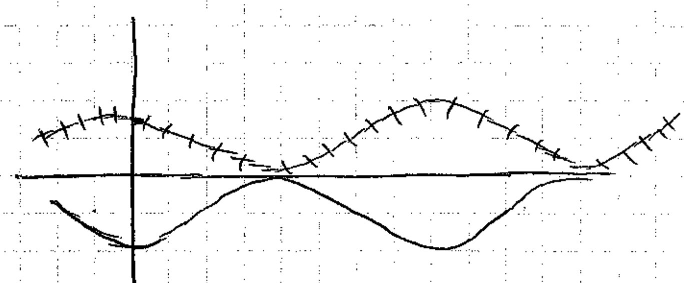
1. Simplify: $\sin^2 x \sec^2 x - \sin^2 x$

$= \sin^2 x \cdot \frac{1}{\cos^2 x} - \sin^2 x$

$= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$

$= \tan^2 x - \sin^2 x$

2. Verify: $\sin^2 x - 1 = \cos^2 x$



No; —— $= \sin^2 x - 1$

++++ $= \cos^2 x$

3. Prove: $\frac{1}{\cos x} - \cos x = \frac{\tan x}{\csc x}$

$= \frac{1 - \cos^2 x}{\cos x}$

$= \frac{\sin^2 x}{\cos x}$

$= \frac{\sin x}{\cos x} \cdot \sin x$

$= \tan x \cdot \sin x$

$\cancel{\frac{\sin x}{\cos x}} \cancel{\cdot \frac{1}{\sin x}}$

$\cancel{= \frac{1}{\cos x}} \cancel{\sin x}$

$\frac{\sin x}{\cos x} \times \frac{1}{\csc x}$

$= \tan x \times \sin x$

4. $\frac{\sec x}{1 - \cos x} = \frac{\sec x + 1}{\sin^2 x}$

$\frac{1}{\cos x} \times \frac{1}{1 - \cos x}$

$= \frac{1}{\cos x - \cos^2 x}$

$\frac{\sec x}{\sin^2 x} + \frac{1}{\sin^2 x}$

$\cancel{\frac{1}{\cos x}} \cdot d$

$\frac{\sec x}{\sin^2 x} + \csc^2 x$

$\frac{\sec x + \sin^2 x \csc^2 x}{\sin^2 x}$

$\frac{\sec x}{1 - \cos x} = \frac{\sec^2 x + 1}{\sin^2 x}$

$\frac{\frac{1}{\cos x} \cdot \cos x}{1 - \cos x \cdot \cos x} = \frac{\left(\frac{1}{\cos x} + 1\right) \cos x}{(\sin^2 x) \cos x}$

$\frac{1}{\cos x (1 - \cos x)} \cdot \frac{(1+\cos x)}{(1+\cos x)} = \frac{1 + \cos x}{\sin^2 x \cdot \cos x}$

$(1 + \cos x)$

$\frac{1 + \cos x}{\cos x (1 - \cos^2 x)}$

$\frac{1 + \cos x}{\cos x \cdot \sin^2 x}$

6.3

1. a) $\sin\left(\frac{\pi}{2}+x\right)$

$= \sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x$

$= \cos x$

b) $\tan\left(x+\frac{\pi}{3}\right)$

$= \dfrac{\tan x + \tan\frac{\pi}{3}}{1-\tan x\tan\frac{\pi}{3}}$

$= \dfrac{\tan x + \sqrt{3}}{1-\sqrt{3}\tan x}$

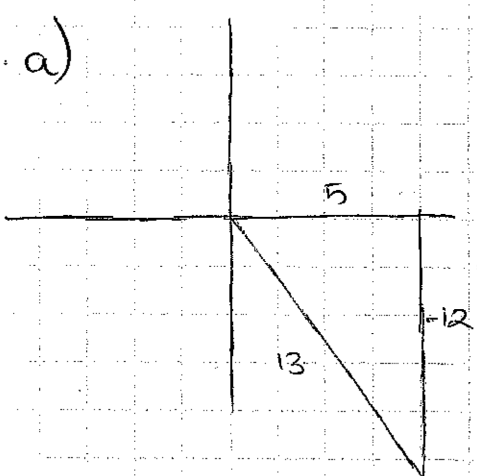
c) $\cos\left(x+\frac{\pi}{4}\right)$

$= \cos x\cos\left(\frac{\pi}{4}\right) - \sin x\sin\left(\frac{\pi}{4}\right)$

$= \dfrac{\cos x}{\sqrt{2}} - \dfrac{\sin x}{\sqrt{2}}$

$= \dfrac{\cos x-\sin x}{\sqrt{2}}$

3. a)



$\sin\left(A+\frac{\pi}{2}\right)$

$= \sin A\cos\frac{\pi}{2} + \cos A\sin\left(\frac{\pi}{2}\right)$

$0 + \frac{5}{13}(1)$

$= \frac{5}{13}$

b) $\tan\left(A+\frac{\pi}{3}\right)$

$= \dfrac{\tan A + \tan\left(\frac{\pi}{3}\right)}{1-\tan A\tan\left(\frac{\pi}{3}\right)}$

$= \dfrac{\frac{-12}{5}+\sqrt{3}}{1-\left(\frac{-12}{5}\right)(\sqrt{3})} = \dfrac{\sqrt{3}-\frac{12}{5}}{1+\frac{12\sqrt{3}}{5}}$

c) $\csc\left(A+\frac{\pi}{2}\right)$

$= \dfrac{1}{\sin\left(A+\frac{\pi}{2}\right)}$

$= \dfrac{1}{\sin A\cos\left(\frac{\pi}{2}\right)+\cos A\sin\left(\frac{\pi}{2}\right)}$

$= \dfrac{1}{\frac{5}{13}} = \frac{13}{5}$

6.4

1a) $\dfrac{\cancel{2}\cancel{\sin x}}{\cancel{2}\cancel{\sin x}\cos x}$

$= \sec x$

b. $\dfrac{\left(\cos^2 x - \left(\cos^2 x - \sin^2 x\right)\right)}{\sin x}$

$= \dfrac{\cancel{\cos^2 x - \cos^2 x} + \sin^2 x}{\cancel{\sin x}}$

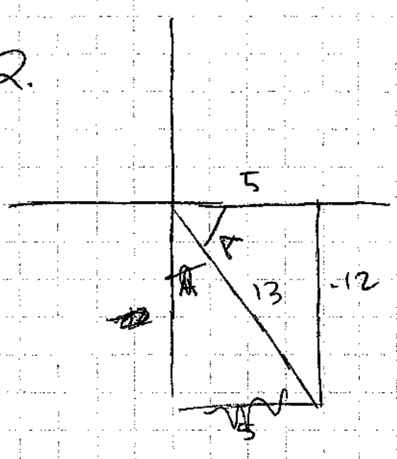
$= \sin x$

c.) $\dfrac{\cancel{8}\tan(10x)}{4 - 4\tan^2(10x)}$

$= \dfrac{2\tan(10x)}{\cancel{4}\,1 - \tan^2(10x)}$

$= \tan 2(10x) = \tan 20x$

2.



a) $\sin(2A) = 2\sin A\cos A$

$= 2\left(\frac{-12}{13}\right)\left(\frac{5}{13}\right)$

$= \frac{-120}{169}$

b) $\csc(2A) = \frac{1}{\sin(2A)} = \frac{-169}{120}$

c) $\cos(2A) = 2\cos^2 A - 1$

$= 2\left(\frac{5}{13}\right)^2 - 1$

$= 2\left(\frac{25}{169}\right) - 1$

$= \frac{50}{169} - \frac{169}{169}$

$= \frac{-119}{169}$

d) $\sec(2A) = \frac{1}{\cos(2A)} = \frac{-169}{119}$

e) $\tan(2A) = \dfrac{2\tan A}{1 - \tan^2 A}$

$= \dfrac{2\left(\frac{-12}{5}\right)}{1 - \left(\frac{-12}{5}\right)^2}$

$= \dfrac{\frac{-24}{5}}{1 - \left(\frac{144}{25}\right)}$

$= \dfrac{\frac{-24}{5}}{\frac{25}{25} - \frac{144}{25}} = \dfrac{\frac{-24}{5}}{\frac{-119}{25}}$

$= \frac{+24}{\cancel{5}} \times \frac{+\cancel{25}^{5}}{119} = \frac{120}{119}$

f) $\cot(2A) = \frac{1}{\tan(2A)} = \frac{119}{120}$

6.4 cont.

Proove: $\sin^2 x - \cos^2 x = 2\sin^2 x - 1$

$$-1(-\sin^2 x + \cos^2 x) = -1(-2\sin^2 x + 1)$$

$$-1(\cos^2 x - \sin^2 x)$$

$$-1(\cos 2x)$$

$$= -\cos(2x)$$

$$-1(1 - 2\sin^2 x)$$

$$-1(\cos 2x)$$

$$= -\cos(2x)$$

4. $\dfrac{2\cos x + 2\cos^2 x}{\sin 2x} = \dfrac{\sin x}{1 - \cos x} \cdot \dfrac{(1+\cos x)}{(1+\cos x)}$

$$\frac{\cancel{2\cos x}(1 + \cos x)}{\cancel{2}\sin x \cancel{\cos x}}$$

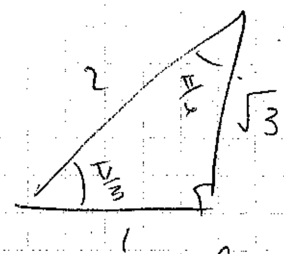
$$= \frac{1 + \cos x}{\sin x}$$

$$\frac{\sin x + \sin x\cos x}{1 - \cos^2 x}$$

$$\frac{\cancel{\sin x}(1 + \cos x)}{\sin^2 x}$$

$$\frac{1 + \cos x}{\sin x}$$

6.5. Restrictions



$\cos\left(\frac{\pi}{6}\right) = \sqrt{}$

1. $\dfrac{\cos x}{1+\sin x}$

$\sin x \neq -1$

$x \neq \text{ArcSin}(-1)$

$x = -\frac{\pi}{2} + 2\pi n$

n= integer

2. $\dfrac{\frac{1}{\cos x}}{1-\cos x}$

$\cos x \neq 0 \qquad \cos x \neq 1$

$x \neq \text{ArcCos}(0) \qquad x \neq \text{ArcCos}(1)$

$x \neq \frac{\pi}{2} + \pi n \qquad x \neq \frac{\pi}{2} + 2\pi n$

n= number

3. $y = \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$

$\sin x \neq 0 \qquad \cos x \neq 0$

$x \neq \text{ArcSin}(0) \qquad x \neq \text{ArcSin}(0)$

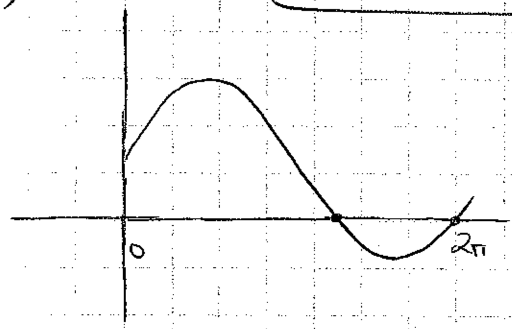
$x \neq 0 + \pi n \qquad x \neq \frac{\pi}{2} + \pi n$

n= number

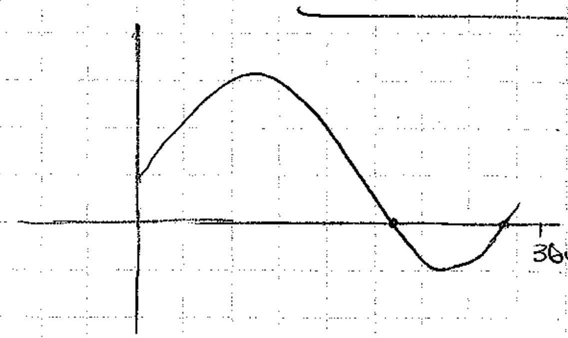
Chapter 6.6

1. $3\sin x + 2 = 0$ (graph it)

a) $0 \leq x \leq 2\pi$ $x = 3.87$ and 5.55



b) $0^\circ \leq x \leq 360^\circ$ $x = 233.62^\circ$ and 318.19°



2. $\tan x = -\sqrt{3}$

a) $x = 2.09$ and 5.24

b) $x = -1.05$ and 2.09

c) $x = -1.05$ and

d) $x = 120^\circ$ and 300°

Chapter 6.67

1. $\sqrt{3}\sec x \sin x - 2\sin x = 0$

$\sin x\left(\sqrt{3}\sec x - 2\right) = 0$

$\sin x = 0 \quad \sqrt{3}\sec x - 2 = 0$

$\sin x = 0 \quad \sec x = 2/\sqrt{3}$

$\frac{1}{\cos x} = \frac{2}{\sqrt{3}}$

$\sin x = 0 \quad \cos x = \sqrt{3}/2$

a) $x = 0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}$

b) $x = -\pi, -\frac{\pi}{6}, 0, \frac{\pi}{6}$

c) $x = -\frac{\pi}{6}, 0, \frac{\pi}{6}$

d) $x = 0^\circ, 30^\circ, 180^\circ, 330^\circ$

$\cancel{2\sin(3x) - \sqrt{3} = 0}$

$\cancel{2\sin(3x) = \sqrt{3}}$

$\cancel{\sin(3x) = \frac{\sqrt{3}}{2}}$

$\cancel{3x = \frac{\text{ArcSin}\left(\sqrt{3}/2\right)}{3}}$

2. $3\cos^2 x - 8\cos x - 3 = 0$

$3\cos^2 x - 9\cos x + \cos x - 3 = 0$

$3\cos x\left(\cos x - 3\right) + 1\left(\cos x - 3\right) = 0$

$\left(3\cos x + 1\right)\left(\cos x - 3\right) = 0$

$3\cos x + 1 = 0 \quad \cos x - 3 = 0$

$\cos x = -\frac{1}{3} \quad \cos x = 3$

N.P

$x = \text{ArcCos}\left(-\frac{1}{3}\right)$

$x = 1.91$ and 5.05

Chapter 6.8

1. $2\sin(3x) - \sqrt{3} = 0$

$2\sin(3x) = \sqrt{3}$

$\sin(3x) = \dfrac{\sqrt{3}}{2}$

$3x = \text{Arc}\sin\left(\frac{\sqrt{3}}{2}\right)$

$x = \dfrac{\text{Arc}\sin\left(\frac{\sqrt{3}}{2}\right)}{3}$

$$\left.\begin{aligned} &= \frac{\pi}{9} + \frac{2\pi}{3}n \\ &= \frac{2\pi}{9} + \frac{2\pi}{3}n \end{aligned}\right\} n = \text{integer}$$

2. $4\cos(5x) - 2 = 0$

$4\cos(5x) = 2$

$\cos(5x) = \frac{1}{2}$

$5x = \text{Arc}\cos\left(\frac{1}{2}\right)$

$x = \dfrac{\text{Arc}\cos\left(\frac{1}{2}\right)}{5}$

$$\left.\begin{aligned} &= \frac{\pi}{15} + \frac{2\pi}{5}n \\ &= \frac{\pi}{3} + \frac{2\pi}{5}n \end{aligned}\right\} n = \text{integer}$$

3. $8\sin x\cos x - 4 = 0$

$4(2\sin x\cos x - 1) = 0$

$4(\sin(2x) - 1) = 0$

$4\sin 2x - 4 = 0$

$4\sin 2x = 4$

$\sin 2x = 1$

$2x = \text{Arc}\sin(1)$

$x = \dfrac{\text{Arc}\sin(1)}{2}$

$= \dfrac{\pi}{4} + \pi n \quad n = \text{integer}$

4. $\sin x = \cos(2x)$

$\sin x = 1 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x + 2\sin x - \sin x - 1$

$2\sin x(\sin x + 1) - 1(\sin x + 1)$

$(\sin x + 1)(2\sin x - 1)$

$\sin x + 1 = 0 \quad 2\sin x - 1 = 0$

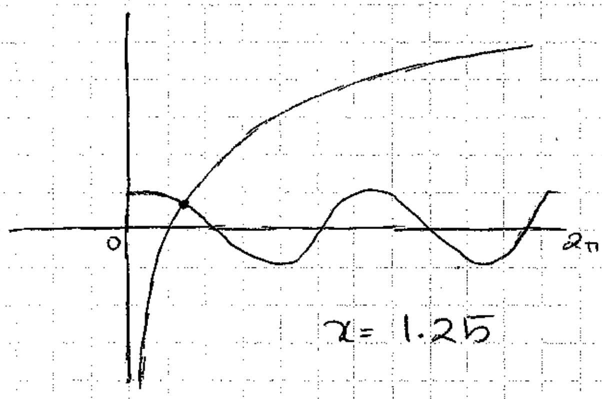
$\sin x = -1 \quad \sin x = +\frac{1}{2}$

$\frac{3\pi}{2} \cancel{+2\pi n}$ $\quad \frac{\pi}{6} \cancel{+2\pi n}$

~~n = integer~~ $\quad \frac{5\pi}{6} \cancel{+2\pi n}$

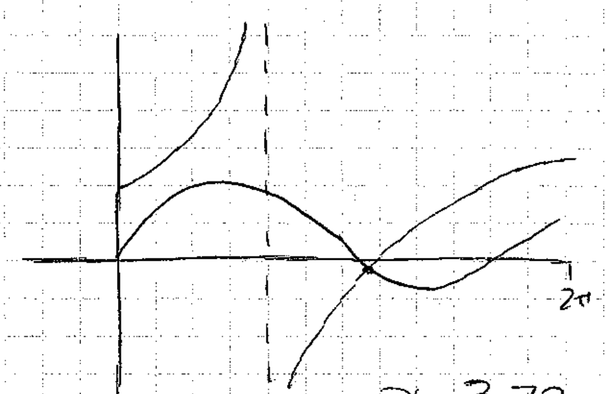
6.9

1. $\log_2(x) = \cos x$



$x = 1.25$

2. $\sec x + \tan x = \sin x$



$x = 3.72$