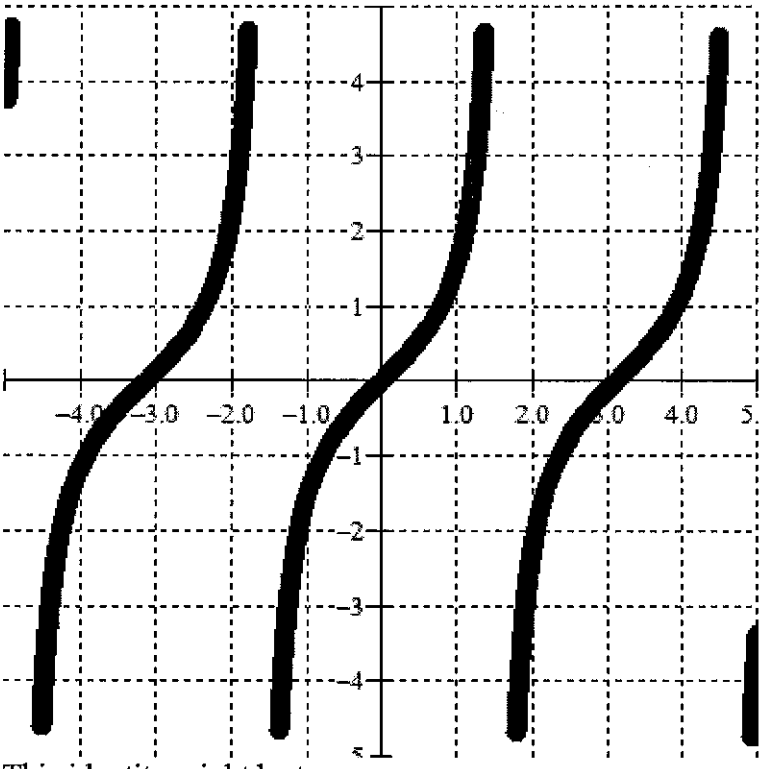
Chapter 6 Answer Key

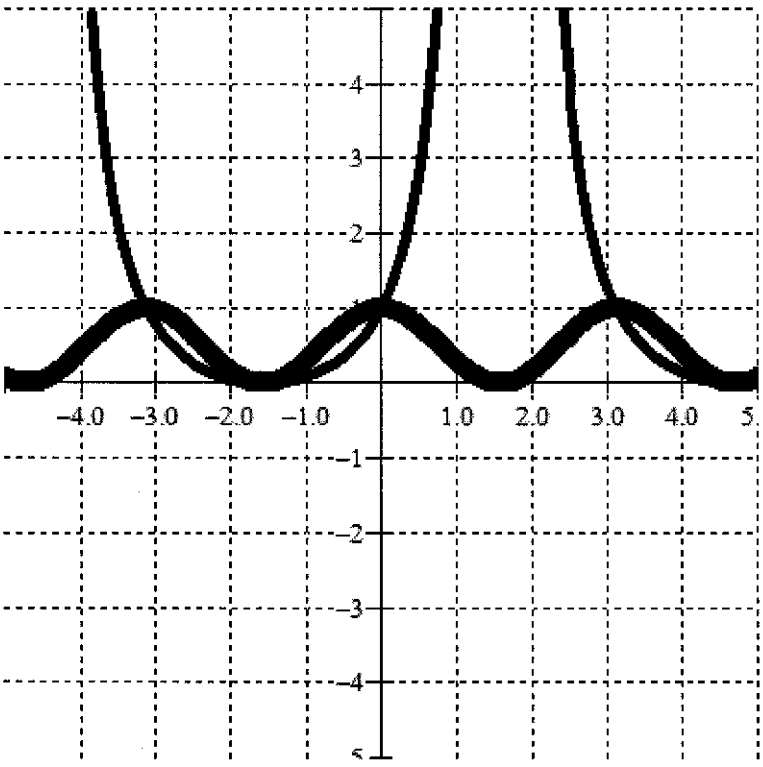
6.1 Reciprocal Identities

1a.



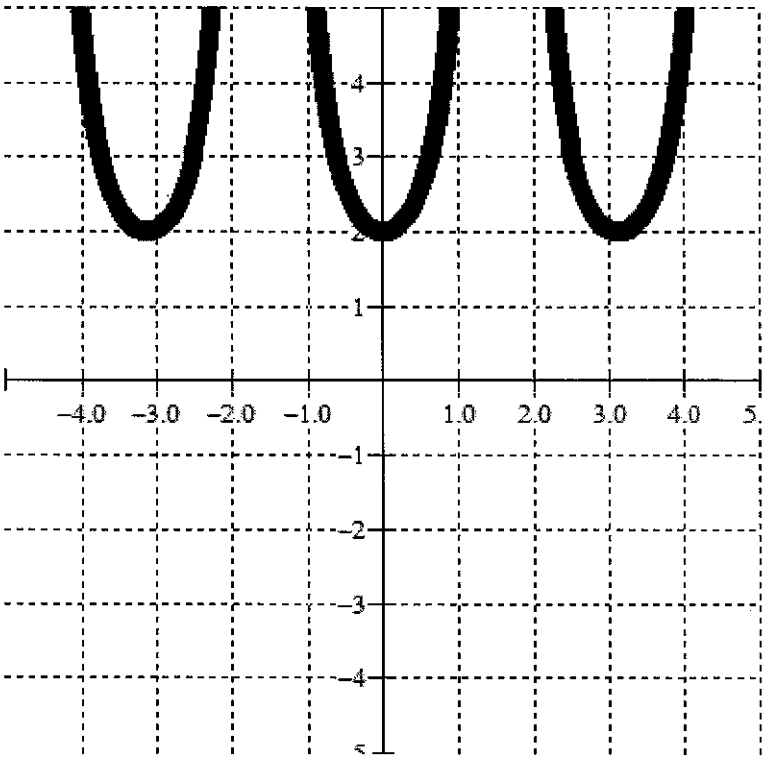
This identity might be true.

(graphs are right on top of each other



1.b

This identity isn't true



1.c

This identity might be true

graphs are the same.

Section 6.1

2. a) $\tan\theta\cos\theta = \sin\theta$

$\left(\frac{\sin\theta}{\cos\theta}\right)\cdot\cos\theta = \sin\theta$

$\sin\theta = \sin\theta$

Q.E.D.

b) $\cot\theta\sec\theta = \csc\theta$

$\left(\frac{\cos\theta}{\sin\theta}\right)\left(\frac{1}{\cos\theta}\right) = \left(\frac{1}{\sin\theta}\right)$

$\frac{1}{\sin\theta} = \frac{1}{\sin\theta}$

Q.E.D

c) $\tan\theta\csc\theta = \sec\theta$

$\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right) = \frac{1}{\cos\theta}$

$\frac{1}{\cos\theta} = \frac{1}{\cos\theta}$

Q.E.D

d) $\sin\theta = \frac{\tan\theta}{\sec\theta}$

$\sin\theta = \left(\frac{\sin\theta}{\cos\theta}\right)\cdot\cos\theta$

$\sin\theta = \sin\theta$

Q.E.D

e) $\csc\theta(1+\sin\theta) = 1+\csc\theta$

$\csc\theta + \csc\theta\sin\theta = 1+\csc\theta$

$\left(\frac{1}{\sin\theta}\right) + \left(\frac{1}{\sin\theta}\right)\cdot\sin\theta = 1+\frac{1}{\sin\theta}$

$1+\frac{1}{\sin\theta} = 1+\frac{1}{\sin\theta}$

Q.E.D

f) $\sin\theta(1+\csc\theta) = 1+\sin\theta$

$\sin\theta + \sin\theta\csc\theta = 1+\sin\theta$

$\sin\theta + \sin\theta\left(\frac{1}{\sin\theta}\right) = 1+\sin\theta$

$1+\sin\theta = 1+\sin\theta$

Q.E.D

g) $\sin\theta\sec\theta\cot\theta = 1$

$(\sin\theta)\left(\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) = 1$

$1 = 1$

Q.E.D

h) $\cot\theta = \frac{1+\cot\theta}{1+\tan\theta}$

$\frac{\cos\theta}{\sin\theta} = \frac{\left(1+\frac{\cos\theta}{\sin\theta}\right)\cdot\sin\theta\cos\theta}{\left(1+\frac{\sin\theta}{\cos\theta}\right)\cdot\sin\theta\cos\theta}$

$= \frac{\sin\theta+\cos\theta}{\sin\theta} \div \frac{\cos\theta+\sin\theta}{\cos\theta}$

$= \frac{\sin\theta+\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta+\sin\theta}$

$\frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}$

Q.E.D

6.1 continued...

2. i) $\sin\theta\tan\theta+\sec\theta=\dfrac{\sin^2\theta+1}{\cos\theta}$

$\sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\right)+\left(\dfrac{1}{\cos\theta}\right)$

$\dfrac{\sin^2\theta+1}{\cos\theta}=\dfrac{\sin^2\theta+1}{\cos\theta}$

Q.E.D

j) $\dfrac{1+\cos\theta}{1-\cos\theta}=\dfrac{1+\sec\theta}{\sec\theta-1}$

$\dfrac{\left(1+\frac{1}{\cos\theta}\right)\cdot\cos\theta}{\left(\frac{1}{\cos\theta}-1\right)\cdot\cos\theta}$

$\dfrac{\left(\frac{\cos\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)}{\left(\frac{1}{\cos\theta}-\frac{\cos\theta}{\cos\theta}\right)}$

$\dfrac{\left(\frac{1+\cos\theta}{\cos\theta}\right)}{\left(\frac{1-\cos\theta}{\cos\theta}\right)}$

$\dfrac{1+\cos\theta}{\cancel{\cos\theta}}\cdot\dfrac{\cancel{\cos\theta}}{1-\cos\theta}$

$\dfrac{1+\cos\theta}{1-\cos\theta}=\dfrac{1+\cos\theta}{1-\cos\theta}$

Q.E.D

$\dfrac{1+\sin\theta}{1-\sin\theta}=\dfrac{\csc\theta+1}{\csc\theta-1}$

$\dfrac{\left(\frac{1}{\sin\theta}+1\right)}{\left(\frac{1}{\sin\theta}-1\right)}$

$\dfrac{\left(\frac{1}{\sin\theta}+\frac{\sin\theta}{\sin\theta}\right)}{\left(\frac{1}{\sin\theta}-\frac{\sin\theta}{\sin\theta}\right)}$

$\dfrac{\left(\frac{1+\sin\theta}{\sin\theta}\right)}{\left(\frac{1-\sin\theta}{\sin\theta}\right)}$

$\dfrac{1+\sin\theta}{1-\sin\theta}\cdot\dfrac{1+\sin\theta}{1+\sin\theta}$

$\dfrac{1+\sin\theta}{1-\sin\theta}$

Q.E.D

$\dfrac{1+\tan\theta}{1+\cot\theta}=\dfrac{1-\tan\theta}{\cot\theta-1}$

$\dfrac{1+\tan\theta}{1+\frac{1}{\tan\theta}}$

$\dfrac{1+\tan\theta}{\left(\frac{\tan\theta+1}{\tan\theta}\right)}$

$\dfrac{1-\tan\theta}{\frac{1}{\tan\theta}-1}$

$\dfrac{1-\tan\theta}{\left(\frac{1-\tan\theta}{\tan\theta}\right)}$

$\tan\theta=\tan\theta$

Q.E.D.

Chapter 6.1

2k.)

$$\frac{\sin\theta + \tan\theta}{\cos\theta + 1} = \tan\theta$$

$$= \frac{\sin\theta + \frac{\sin\theta}{\cos\theta}}{\cos\theta + 1} = \tan\theta$$

$$= \frac{\frac{\sin\theta\cos\theta + \sin\theta}{\cos\theta}}{\cos\theta + 1} = \tan\theta$$

$$= \frac{\sin\theta(\cos\theta + 1)}{\cos\theta} \cdot \frac{1}{\cos\theta + 1} = \tan\theta$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\tan\theta = \tan\theta$$

Chapter 6.1 cont

no number) $\frac{1+\sin\theta}{1+\csc\theta}$ = $\sin\theta$

$\frac{1+\sin\theta}{\left(\frac{\sin\theta}{\sin\theta}+\frac{1}{\sin\theta}\right)}$

$\frac{1+\sin\theta}{\left(\frac{\sin\theta+1}{\sin\theta}\right)}$

$\sin\theta$ = $\sin\theta$

k $\frac{\sin\theta+\tan\theta}{\cos\theta+1}$ = $\tan\theta$

$\frac{\left(\sin\theta+\left(\frac{\sin\theta}{\cos\theta}\right)\right)}{\cos\theta+1}$

$\frac{\left(\frac{\sin\theta\cos\theta+\sin\theta}{\cos\theta}\right)}{\cos\theta+1}$

$\frac{\left(\frac{\sin\theta(\cos\theta+1)}{\cos\theta}\right)}{\cos\theta+1}$

$\frac{\sin\theta}{\cos\theta}$ = $\frac{\sin\theta}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta}$

Q.E.D.

Chapter 6.2

1.a) $\sin^2\theta$ = $\frac{\tan^2\theta}{1+\tan^2\theta}$

$\frac{\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}{\sec^2\theta}$

$\frac{\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}{\left(\frac{1}{\cos^2\theta}\right)}$

$\sin^2\theta$ = $\sin^2\theta$

Q.E.D

b) $\csc^2\theta - 1$ = $\csc^2\theta\cos^2\theta$

$\left(\frac{1}{\sin^2\theta}\right)\cos^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta}$

$\cot^2\theta$

$\csc^2\theta - 1$ = $\csc^2\theta - 1$

Q.E.D

Chapter 6.2 cont

c) $\frac{\tan\theta}{\sec\theta+1} = \frac{\sec\theta-1}{\tan\theta}$

$\frac{\sec\theta-1}{\tan\theta} \cdot \frac{\sec\theta+1}{\sec\theta+1}$

$\frac{\sec^2\theta-1}{\tan\theta(\sec\theta+1)}$

$\frac{\tan^2\theta}{\tan\theta(\sec\theta+1)}$

$\frac{\tan\theta}{\sec\theta+1} = \frac{\tan\theta}{\sec\theta+1}$

Q.E.D

d) $\frac{\sin\theta+\cos\theta\cot\theta}{\cot\theta} = \sec\theta$

$\frac{\sin\theta+\cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{\cos\theta}{\sin\theta}\right)}$

$\frac{\left(\sin\theta+\frac{\cos^2\theta}{\sin\theta}\right)}{\left(\frac{\cos\theta}{\sin\theta}\right)}$

$\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta}\right)\frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} = \frac{1}{\cos\theta}$

Q.E.D

e) $\sin\theta\cos\theta\tan\theta = 1-\cos^2\theta$

$\sin\theta\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)$ → $\sin^2\theta$

$\sin^2\theta = \sin^2\theta$

Q.E.D

f) $\frac{\sin\theta+\cos\theta}{\csc\theta+\sec\theta} = \sin\theta\cos\theta$

$\frac{\sin\theta+\cos\theta}{\left(\frac{1}{\sin\theta}+\frac{1}{\cos\theta}\right)}$

$\frac{\sin\theta+\cos\theta}{\left(\frac{\cos\theta+\sin\theta}{\cos\theta\sin\theta}\right)}$

$\cos\theta\sin\theta = \sin\theta\cos\theta$

Q.E.D

g) $\tan^2\theta(1+\cot^2\theta) = \sec^2\theta$

$\tan^2\theta\cdot\csc^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{1}{\sin^2\theta}$

$\frac{1}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

Q.E.D

h) $\tan\theta+\cot\theta = \sec\theta\csc\theta$

$\left(\frac{\sin\theta}{\cos\theta}\right)+\left(\frac{\cos\theta}{\sin\theta}\right)$ $\left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$

$\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$

$\frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

Q.E.D

Chapter 6.2 cont

i) $\sec^2\theta + \csc^2\theta = \sec^2\theta \csc^2\theta$

$\left(\frac{1}{\cos^2\theta}\right) + \left(\frac{1}{\sin^2\theta}\right) \quad \left(\frac{1}{\cos^2\theta}\right)\left(\frac{1}{\sin^2\theta}\right)$

$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta} \quad \frac{1}{\cos^2\theta\sin^2\theta}$

$\frac{1}{\cos^2\theta\sin^2\theta} = \frac{1}{\cos^2\theta\sin^2\theta}$

Q.E.D

j) $\sin^2\theta = \cos\theta(\sec\theta - \cos\theta)$

$1-\cos^2\theta \quad \cos\theta\left(\frac{1}{\cos\theta} - \cos\theta\right)$

$\cos\theta\left(\frac{1-\cos^2\theta}{\cos\theta}\right)$

$1-\cos^2\theta = 1-\cos^2\theta$

Q.E.D

k) $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$

$\frac{\sin^2 x}{\sin x(1+\cos x)} + \frac{(1+2\cos x+\cos^2 x)}{\sin x(1+\cos x)}$

$\frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$

$\frac{2+2\cos x}{\sin x(1+\cos x)}$

$\frac{2(1+\cos x)}{\sin x(1+\cos x)}$

$\frac{2}{\sin x}$

$2\csc x = 2\csc x$

Q.E.D

l) $\frac{1+\csc x}{\cot x + \cos x} = \frac{\cot x}{\csc x - \sin x}$

$\frac{\left(1+\frac{1}{\sin x}\right)}{\left(\frac{\cos x}{\sin x}\right)+\cos x} \quad \frac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{1}{\sin x} - \sin x\right)}$

$\frac{\left(\frac{\sin x + 1}{\sin x}\right)}{\left(\frac{\cos x + \cos x\sin x}{\sin x}\right)} \quad \frac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{1-\sin^2 x}{\sin x}\right)}$

$\frac{\sin x + 1}{\cos x + \cos x\sin x} \quad \frac{\cos x}{\cos^2 x}$

$\frac{\sin x + 1}{\cos x(1+\sin x)} \quad \frac{1}{\cos x}$

$\frac{1}{\cos x} = \frac{1}{\cos x}$

Q.E.D

m) $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

$\frac{1-\sin\theta + 1 + \sin\theta}{1-\sin^2\theta}$

$\frac{2}{\cos^2\theta}$

$2\sec^2\theta = 2\sec^2\theta$

Q.E.D

Chapter 6.3

1. a) $\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)$

$= \sin\frac{\pi}{6}\cos\frac{\pi}{3} + \cos\frac{\pi}{6}\sin\frac{\pi}{3}$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{4} + \frac{3}{4} = \boxed{1}$

b) ~~tan~~ $\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{6} - \cos\frac{\pi}{3}\sin\frac{\pi}{6}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

$$\frac{\tan\left(\frac{\pi}{3}\right)-\tan\left(\frac{\pi}{6}\right)}{1+\tan\left(\frac{\pi}{3}\right)\left(\tan\left(\frac{\pi}{6}\right)\right)}$$

$$\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+1} = \frac{\frac{3-1}{\sqrt{3}}}{2}$$

$$= \frac{1}{\sqrt{3}}$$

c) $\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)$

$= \cos\frac{\pi}{2}\cos\frac{\pi}{6} - \sin\frac{\pi}{2}\sin\frac{\pi}{6}$

$= (0)\left(\frac{\sqrt{3}}{2}\right) - (1)\left(\frac{1}{2}\right)$

$= \boxed{-\frac{1}{2}}$

2. $\cos\frac{7\pi}{12} = \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \left(\frac{1}{2\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2\sqrt{2}}\right) = \boxed{\frac{1-\sqrt{3}}{2\sqrt{2}}}$

3. a) $\sin\left(\frac{\pi}{2}+\emptyset\right) = \cos\emptyset$

$\sin\frac{\pi}{2}\cos\emptyset + \cos\frac{\pi}{2}\sin\emptyset$

$(1)\cos\emptyset + (0)\sin\emptyset$

$\cos\emptyset = \cos\emptyset$

Q.E.D.

b) $\cos\left(\frac{3\pi}{2}+\emptyset\right) = \sin\emptyset$

$\cos\frac{3\pi}{2}\cos\emptyset - \sin\frac{3\pi}{2}\sin\emptyset$

$(0)\cos\emptyset - (-1)\sin\emptyset$

$\sin\emptyset = \sin\emptyset$

Q.E.D

4. a) $\cos(\pi-\emptyset)$

$= \cos\pi\cos\emptyset + \sin\pi\sin\emptyset$

$= (-1)\cos\emptyset + (0)(\sin\emptyset)$

$= \boxed{-\cos\emptyset}$

b) $\sin(\pi-x)$

$= \sin\pi\cos x - \cos\pi\sin x$

$= (0)\cos x - (-1)\sin x$

$= \boxed{\sin x}$

Chapter 6.3 cont.

c) $\tan(\pi + 2x)$ $\quad \frac{\tan\pi + \tan 2x}{1 - \tan\pi + \tan 2x} = \frac{\tan 2x}{1 + \tan 2x}$

= [illegible]

= [illegible]

= $\sin 2x$

5. a) $\sin x \cos 5x + \cos x \sin 5x$

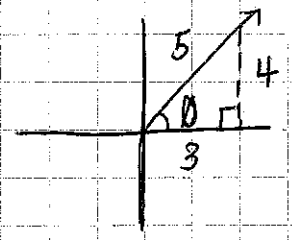
= $\boxed{\sin(x+5x)} - \sin 6x$

b) $\cos x \cos 8x - \sin x \sin 8x$

= $\cos(x+8x) = \boxed{\cos 9x}$

→ see attachment for c, d

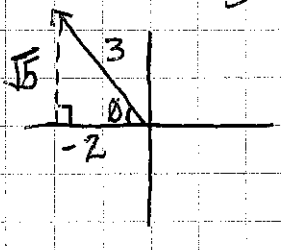
6. $\sin\theta = \frac{4}{5}$ $\quad \sin(\theta + \frac{\pi}{6}) = \sin\theta\cos\frac{\pi}{6} + \cos\theta\sin\frac{\pi}{6}$



$= \left(\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{3}{5}\right)\left(\frac{1}{2}\right)$

$= \left(\frac{4\sqrt{3}}{10}\right) + \left(\frac{3}{10}\right) = \boxed{\frac{4\sqrt{3}+3}{10}}$

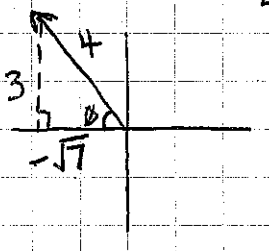
7. $\cos\theta = -\frac{2}{3}$ $\quad \sin(\theta + \frac{\pi}{4}) = \sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}$



$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{2}{3}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{5}}{3\sqrt{2}} - \frac{2}{3\sqrt{2}} = \boxed{\frac{\sqrt{5}-2}{3\sqrt{2}}}$

8. $\sin\theta = 0.75 = \frac{3}{4}$ $\quad \sin(\theta + \frac{\pi}{3}) = \sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3}$



$= \left(\frac{3}{4}\right)\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{7}}{4}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \left(\frac{3}{8}\right) - \left(\frac{\sqrt{21}}{8}\right) = \boxed{\frac{3-\sqrt{21}}{8}}$

Chapter 6.3 cont.

9. $\sin\left(\frac{\pi}{4}+\theta\right)+\sin\left(\frac{\pi}{4}-\theta\right) = \sqrt{2}\cos\theta$

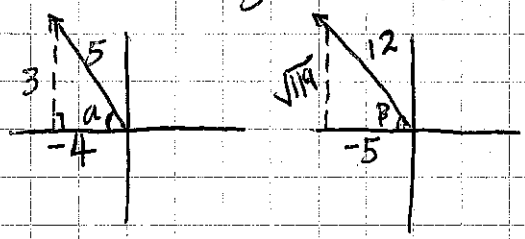
RHS $\left(\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta\right)+\left(\sin\frac{\pi}{4}\cos\theta-\cos\frac{\pi}{4}\sin\theta\right)$

$\left(\left(\frac{1}{\sqrt{2}}\right)(\cos\theta)+\left(\frac{1}{\sqrt{2}}\right)(\sin\theta)\right)+\left(\left(\frac{1}{\sqrt{2}}\right)(\cos\theta)-\left(\frac{1}{\sqrt{2}}\right)(\sin\theta)\right)$

$\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\cos\theta = 2\left(\frac{1}{\sqrt{2}}\right)\cos\theta \rightarrow \frac{\sqrt{2}}{\sqrt{2}}\cdot\frac{2}{\sqrt{2}}\cos\theta = \frac{2\sqrt{2}}{2}\cos\theta = \boxed{\sqrt{2}\cos\theta}$

$\therefore$ RHS = LHS $\sqrt{2}\cos\theta = \sqrt{2}\cos\theta$

10. $\sin\alpha = \frac{3}{5}$ $\cos\beta = -\frac{5}{12}$



a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta$

$= \left(\frac{3}{5}\right)\left(-\frac{5}{12}\right)+\left(\frac{4}{5}\right)\left(\frac{\sqrt{119}}{12}\right)$

$= -\frac{15}{60}-\frac{4\sqrt{119}}{60}$

$= \boxed{\frac{-4\sqrt{119}-15}{60}}$

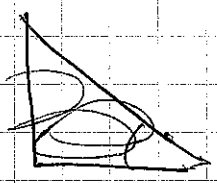
b) $\cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta$

$= \left(-\frac{4}{5}\right)\left(-\frac{5}{12}\right)+\left(\frac{3}{5}\right)\left(\frac{\sqrt{119}}{12}\right)$

$= +\frac{20}{60}+\frac{3\sqrt{119}}{60} = \boxed{\frac{3\sqrt{119}+20}{60}}$

10.) c. $\tan(\alpha+\beta)$

~~b tan $\frac{3}{5}$ + tan $\frac{5}{12}$~~



$\tan\alpha = \frac{3}{4}$ $\tan\beta = \frac{\sqrt{119}}{-5}$

10c

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \times \tan\beta}$$

$$= \frac{-\frac{3}{4} \times \frac{\sqrt{119}}{-5}}{1 - \left(\frac{-3}{4}\right) \times \left(\frac{\sqrt{119}}{-5}\right)} = \frac{\frac{3}{20}\sqrt{119}}{1 - \frac{3\sqrt{119}}{20}}$$

$$= \frac{3\sqrt{119}}{1 \times 5}$$

Section 6.4

1. a) $2\sin 0.6\cos 0.6$
$= \sin(2 \cdot 0.6)$
$= \boxed{\sin 1.2}$

b) $2\sin 3\cos 3$
$= \sin(2 \cdot 3)$
$= \boxed{\sin 6}$

c) $2\sin 2\cos 2$
$= \sin(2 \cdot 2)$
$= \boxed{\sin 4}$

k) $\dfrac{2\tan 5}{1-\tan^2 5} = \tan(10)$

d) $\cos^2 0.45 - \sin^2 0.45$
$= \cos(2 \cdot 0.45)$
$= \boxed{\cos 0.9}$

e) $2\cos^2 5 - 1$
$= \cos(2 \cdot 5)$
$= \boxed{\cos 10}$

f) $1 - 2\sin^2 3$
$= \cos(2 \cdot 3)$
$= \boxed{\cos 6}$

l) $\dfrac{4\tan(6x)}{1-\tan^2 6x} = 2\tan(12x)$

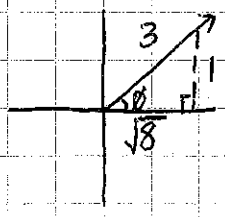
g) $2\sin\frac{\pi}{6}\cos\frac{\pi}{6}$
$= \sin(2 \cdot \frac{\pi}{6})$
$= \sin\frac{2\pi}{6} = \sin\frac{\pi}{3}$
$= \boxed{\frac{\sqrt{3}}{2}}$

h) $\cos^2\frac{\pi}{10} - \sin^2\frac{\pi}{10}$
$= \cos(2 \cdot \frac{\pi}{10})$
$= \cos\left(\frac{2\pi}{10}\right)$
$= \boxed{\cos\frac{\pi}{5}}$

i) $2\cos^2\frac{\pi}{2} - 1$
$= \cos(2 \cdot \frac{\pi}{2})$
$= \cos\pi$
$= \boxed{-1}$

m) $\dfrac{10\tan 8x}{5 - 5\tan^2 8x}$
$= \dfrac{5 \cdot 2\tan 8x}{5(1-\tan^2 8x)}$
$= \tan(16x)$

2. $\sin\theta = \frac{1}{3}$



a) $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{1}{3}\right)\left(\frac{\sqrt{8}}{3}\right)$
$= \frac{2\sqrt{8}}{9} = \frac{2 \cdot \sqrt{4} \cdot \sqrt{2}}{9} = \boxed{\frac{4\sqrt{2}}{9}}$

b) $\cos 2\theta = 1 - 2\sin^2\theta$
$= 1 - 2\left(\frac{1}{3}\right)^2 = 1 - 2\left(\frac{1}{9}\right) = 1 - \frac{2}{9} = \boxed{\frac{7}{9}}$

3. a) $\sin(2x - \pi)$
$= \sin 2x\cos\pi - \cos 2x\sin\pi$
$= \sin 2x(-1) - \cos 2x(0)$
$= -\sin 2x$
$= \boxed{-2\sin x\cos x}$

b) $\cos(2x + \pi)$
$= \cos 2x\cos\pi + \sin 2x\sin\pi$
$= \cos 2x \cdot (-1) + \sin 2x(0)$
$= -\cos 2x$
$= -(2\cos^2 x - 1)$ or $-(\cos^2 x - \sin^2 x)$
or $-(1 - 2\sin^2 x)$

6.4 Continued.

3c) $\tan(\pi + 2x) = \dfrac{\tan\pi + \tan 2x}{1 - \tan\pi + \tan(2x)}$

$= \dfrac{0 + \tan(2x)}{1 - 0 + \tan(2x)} = \dfrac{\tan(2x)}{1 + \tan(2x)}$

$= \left(\dfrac{2\tan(x)}{1 - \tan^2 x}\right) \Big/ \left(1 + \dfrac{2\tan x}{1 - \tan^2 x}\right)$

$= \dfrac{2\tan(x)}{1 - \tan^2 x} \div \left(\dfrac{1 - \tan^2 x + 2\tan x}{1 - \tan^2 x}\right)$

$= \dfrac{2\tan(x)}{1 - \tan^2 x} \times \dfrac{1 - \tan^2 x}{1 - \tan^2 x + 2\tan x}$

$= \dfrac{2\tan(x)}{1 - \tan^2 x + 2\tan x}$

Section 6.4 continued...

4. a) $10\sin 5x\cos 5x$

$= (5\times 2)\sin 5x\cos 5x$

$= 5(\sin(2\cdot 5x))$

$= \boxed{5\sin 10x}$

b) $14\cos^2 3x - 14\sin^2 3x$

$= 14(\cos^2 3x - \sin^2 3x)$

$= (7\times 2)\cos^2 3x - \sin^2 3x$

$= 14(\cos(2\cdot 3x))$

$= \boxed{14\cos 6x}$

5. a) $1+\sin 2\theta = (\sin\theta+\cos\theta)^2$

$1+2\sin\theta\cos\theta \quad (\sin\theta+\cos\theta)(\sin\theta+\cos\theta)$

$\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$

$1+2\sin\theta\cos\theta = 1+2\sin\theta\cos\theta$

Q.E.D.

b) $\sin 2\theta = 2\cot\theta\sin^2\theta$

$2\cos\theta\sin\theta \quad 2\left(\frac{\cos\theta}{\sin\theta}\right)\sin^2\theta$

$2\cos\theta\sin\theta = 2\cos\theta\sin\theta$

Q.E.D

$\sin 2\theta = \sin 2\theta$

c) $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$\cos^2\theta-\sin^2\theta \quad \dfrac{1-\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}{1+\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}$

$\dfrac{\left(\frac{\cos^2\theta-\sin^2\theta}{\cancel{\cos^2\theta}}\right)}{\left(\frac{\cos^2\theta+\sin^2\theta}{\cancel{\cos^2\theta}}\right)}$

$\cos^2\theta-\sin^2\theta = \cos^2\theta-\sin^2\theta$

Q.E.D

$\cos 2\theta = \cos 2\theta$

Section 6.4 continued...

d) $\sec^2\theta = \frac{2}{1+\cos 2\theta}$

$\frac{1}{\cos^2\theta} = \frac{2}{1+2\cos^2\theta-1}$

$\frac{1}{\cos^2\theta} = \frac{2}{2\cos^2\theta}$

$\frac{1}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ $\quad \sec^2\theta = \sec^2\theta$

Q.E.D

g) $\sin 4x = 8\sin x\cos^3 x - 4\sin x\cos x$

$\sin(2\cdot 2x) = 4\sin x\cos x(2\cos^2 x - 1)$

$2\sin 2x\cos 2x = 4\sin x\cos x\cdot\cos 2x$

$2(2\sin x\cos x)\cdot\cos 2x$

$2\sin 2x\cos 2x = 2\sin 2x\cos 2x$ $\quad \sin 4x = \sin 4x$

Q.E.D

f) $\frac{2}{1+\cos 2x} = \tan^2 x + 1$

$\frac{2}{1+2\cos^2 x-1} = \sec^2 x$

$\frac{1}{\cos^2 x} = \sec^2 x$

$\sec^2 x = \sec^2 x$

Q.E.D

e) g) $\cos 2x = \frac{1-\tan^2 x}{\sec^2 x}$

$\cos^2\theta - \sin^2\theta = \frac{1-\tan^2 x}{1+\tan^2 x}$

$\frac{1-\left(\frac{\sin^2 x}{\cos^2 x}\right)}{1+\left(\frac{\sin^2 x}{\cos^2 x}\right)}$

$\frac{\left(\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\right)}{\left(\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\right)}$

$\cos^2\theta - \sin^2\theta = \cos^2\theta - \sin^2\theta$ $\quad \cos 2x = \cos 2x$

Section ~~5.4~~ 6.5

1. a) $\frac{2x+1}{x-3}$ $\boxed{x \neq 3}$ b) $\frac{x+3}{(x+3)(x-2)}$ $\boxed{x \neq -3,\ x \neq 2}$

c) $\frac{4x}{x^2+5x+6} = \frac{4x}{(x+3)(x+2)}$ $\boxed{x \neq -3,\ x \neq -2}$

d) $\frac{x+1}{\left(\frac{x-3}{x+4}\right)} = x+1 \cdot \left(\frac{x+4}{x-3}\right)$ $\boxed{x \neq 3,\ x \neq -4}$

2. a) $\sin x \cot x = \sin x \left(\frac{\cos x}{\sin x}\right)$ $\sin x \neq 0$ $\boxed{x \neq 0, \pi}$

b) $\cot x = \frac{\cos x}{\sin x}$ $\sin x \neq 0$ $\boxed{x \neq 0, \pi}$

c) $\frac{\csc x}{\cos x} = \left(\frac{1}{\sin x}\right) \cdot \frac{1}{\cos x}$ $\sin x \neq 0,\ \cos x \neq 0$ $\boxed{x \neq 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}}$

d) $\frac{\cot x}{1-\sin x} = \frac{\left(\frac{\cos x}{\sin x}\right)}{1-\sin x}$ $\sin x \neq 0,\ \sin x \neq 1$ $\boxed{x \neq 0, \pi, \frac{\pi}{2}}$

e) $\frac{\cot x}{1-\cos x}$ $\frac{\left(\frac{\cos x}{\sin x}\right)}{1-\cos x}$ $\sin x \neq 0,\ \cos x \neq 1$ $\boxed{x \neq 0, \pi}$

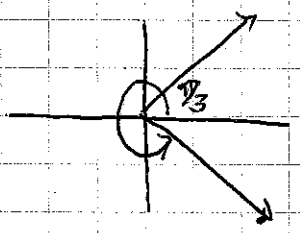
f) $\frac{\tan x}{1-\cos x}$ $\frac{\left(\frac{\sin x}{\cos x}\right)}{1-\cos x}$ $\cos x \neq 0,\ \cos x \neq 1$ $\boxed{x \neq \frac{\pi}{2}, \frac{3\pi}{2}, 0}$

g) $\frac{\sec x}{\sin x - 1}$ $\frac{\left(\frac{1}{\cos x}\right)}{\sin x - 1}$ $\cos x \neq 0,\ \sin x \neq 1$ $\boxed{x \neq \frac{\pi}{2}, \frac{3\pi}{2}}$

h) $\cot x + \tan x = \left(\frac{\cos x}{\sin x}\right) + \left(\frac{\sin x}{\cos x}\right)$ $\sin x \neq 0,\ \cos x \neq 0$ $\boxed{x \neq 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}}$

i) $\frac{1}{2\cos^2 x + \cos x - 1} = \frac{1}{(2\cos x - 1)(\cos x + 1)}$ $\cos x \neq \frac{1}{2},\ \cos x \neq -1$

$\boxed{x \neq \frac{\pi}{3}, \pi}$



$x \neq 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$

Section ~~5.4~~ 6.5 continued...

j) $\dfrac{\sec x}{4\sin^2 x - 1} = \dfrac{\left(\frac{1}{\cos x}\right)}{(2\sin x + 1)(2\sin x - 1)}$

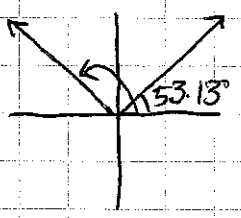
$\cos x \neq 0,\ \sin x \neq -\frac{1}{2},\ \sin x \neq \frac{1}{2}$

$\boxed{x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}}$

k) $\dfrac{1 + 4\sec x}{4 - 5\sin x} = \dfrac{1 + \frac{4}{\cos x}}{4 - 5\sin x}$

$\cos x \neq 0,\ \sin x \neq \frac{4}{5}$

$\boxed{x \neq \frac{\pi}{2}, \frac{3\pi}{2}, 53.13°, 126.87°}$



l) $\dfrac{3 + 2\csc x}{2\sec x - 3} = \dfrac{3 + \frac{2}{\sin x}}{\frac{2}{\cos x} - 3}$ $\left(\dfrac{3\sin x + 2}{\sin x}\right)$ $\dfrac{3\sin x + 2}{\sin x} \cdot \dfrac{\cos x}{2 - 3\cos x}$

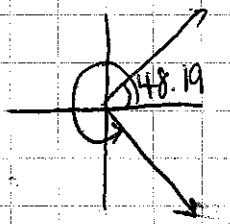
$\left(\dfrac{2 - 3\cos x}{\cos x}\right)$

$\sin x \neq 0,\ \cos x \neq 0$

$\boxed{x \neq 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}}$

$\therefore 2 - 3\cos x \neq 0$

$\cos x \neq \frac{2}{3}$



$\boxed{x \neq 48.19°, 311.81°}$

2. $\dfrac{\sin x}{1 - \sec^2 x} = \dfrac{\sin x}{1 - \frac{1}{\cos^2 x}} = \left(\dfrac{\sin x}{\frac{\cos^2 x - 1}{\cos^2 x}}\right) = \sin x \cdot \dfrac{\cos^2 x}{\cos^2 x - 1}$

① $\cos^2 x \neq 0$

$\cos x \neq 0$

$\boxed{x \neq \frac{\pi}{2}, \frac{3\pi}{2}}$

② $\cos^2 x \neq 1$

$\cos x \neq 1 \quad \cos x \neq -1$

$\boxed{x \neq 0, \pi}$

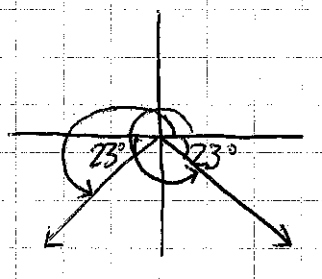
Section ~~5.5~~ 6.6

$0 \le \theta \le 360°$

1. a) $\sin\theta = -0.3926$

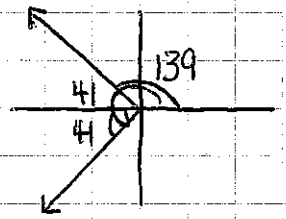
$\theta = -23°$ $\quad \theta_1 = 360 - 23 = \boxed{337°}$

$\theta_2 = 180 + 23 = \boxed{203°}$



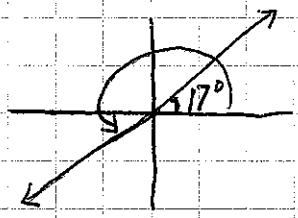
b) $\cos\theta = -0.7515$

$\boxed{\theta_1 = 138.7°}$ $\quad \theta_2 = 180 + 41 = \boxed{221°}$



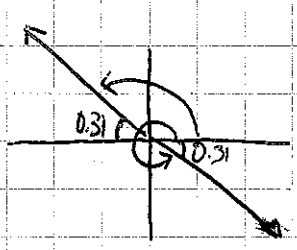
c) $\tan\theta = 0.3125$

$\boxed{\theta_1 = 17°}$ $\quad \theta_2 = 180 + 17 = \boxed{197°}$



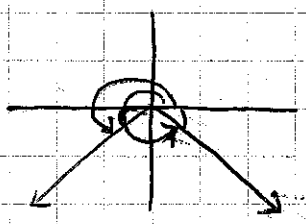
2. a) $\tan\theta = -0.318$ $\quad \theta_1 = \pi - 0.31 = \boxed{2.83}$

$\theta = \boxed{-0.31}$ $\quad \theta_2 = 2\pi - 0.31 = \boxed{5.97}$



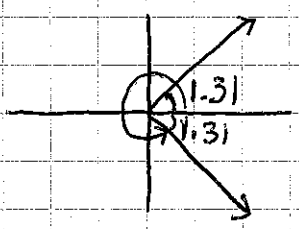
b) $\sin\theta = -0.525$ $\quad \theta_1 = \pi + 0.55 = \boxed{3.69}$

$\theta = -0.55$ $\quad \theta_2 = 2\pi - 0.55 = \boxed{5.73}$



c) $\cos\theta = 0.2599$

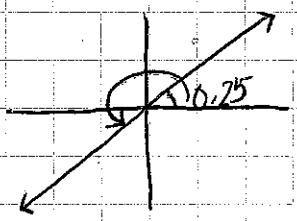
$\boxed{\theta_1 = 1.31}$ $\quad \theta_2 = 2\pi - 1.31 = \boxed{4.97}$



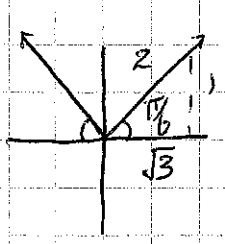
$-\pi \le \theta \le \pi$

3. a) $\tan\theta = 0.254$

$\boxed{\theta_1 = 0.25}$ $\quad \theta_2 = 0.25 - \pi = \boxed{-2.89}$



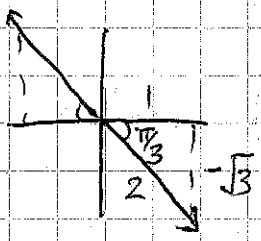
e) $\csc\theta = 2 \quad \sin\theta = \frac{1}{2}$



$\theta_1 = \boxed{\frac{\pi}{6}}$

$\theta_2 = \boxed{\frac{5\pi}{6}}$

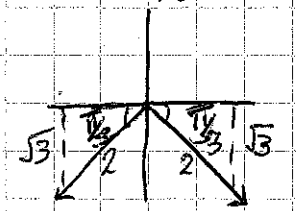
f) $\cot\theta = -\frac{1}{\sqrt{3}} \quad \tan\theta = -\sqrt{3}$



$\theta_1 = \boxed{\frac{2\pi}{3}}$

$\theta_2 = \boxed{\frac{5\pi}{3}}$

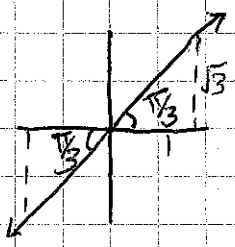
g) $\sin x = -\frac{\sqrt{3}}{2}$



$x_1 = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$

$x_2 = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$

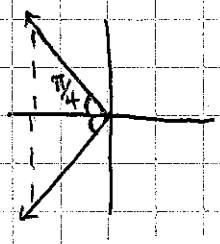
h) $\tan x = \sqrt{3}$



$x_1 = \boxed{\frac{\pi}{3}}$

$x_2 = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$

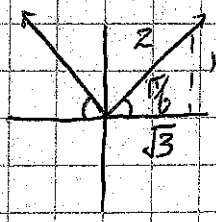
i) $\cos x = -\frac{1}{\sqrt{2}}$



$x_1 = \pi - \frac{\pi}{4} = \boxed{\frac{3\pi}{4}}$

$x_2 = \pi + \frac{\pi}{4} = \boxed{\frac{5\pi}{4}}$

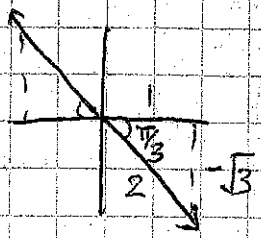
e) $\csc\theta = 2 \quad \sin\theta = \frac{1}{2}$



$\theta_1 = \boxed{\frac{\pi}{6}}$

$\theta_2 = \boxed{\frac{5\pi}{6}}$

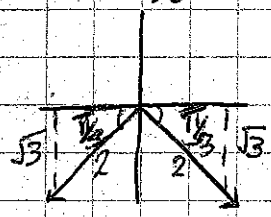
f) $\cot\theta = -\frac{1}{\sqrt{3}} \quad \tan\theta = -\sqrt{3}$



$\theta_1 = \boxed{\frac{2\pi}{3}}$

$\theta_2 = \boxed{\frac{5\pi}{3}}$

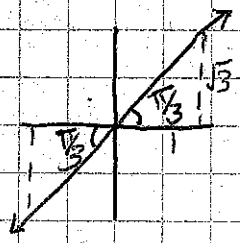
g) $\sin x = -\frac{\sqrt{3}}{2}$



$x_1 = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$

$x_2 = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$

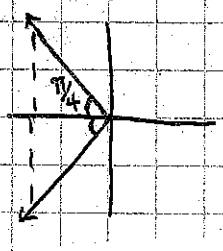
h) $\tan x = \sqrt{3}$



$x_1 = \boxed{\frac{\pi}{3}}$

$x_2 = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$

i) $\cos x = -\frac{1}{\sqrt{2}}$



$x_1 = \pi - \frac{\pi}{4} = \boxed{\frac{3\pi}{4}}$

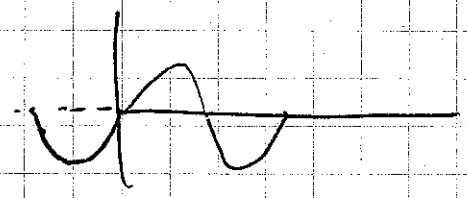
$x_2 = \pi + \frac{\pi}{4} = \boxed{\frac{5\pi}{4}}$

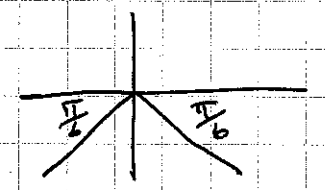
6.7

a) $2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0 \quad \sin x = -\frac{1}{2}$





i) $x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

ii) $x = 0°, 180°, 210°, 330°$

iii) $x = -\pi, 0, -\frac{\pi}{6}, -\frac{5\pi}{6}$

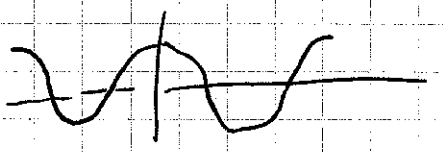
iv) $x = 0, -\frac{\pi}{6}$

---

b) $\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0, \quad \cos x = 1$



i) $x = \frac{\pi}{2}, \frac{3\pi}{2}, 0$

ii) $x = 90°, 270°, 0°$

iii) $x = -\frac{\pi}{2}, \frac{\pi}{2}, 0$

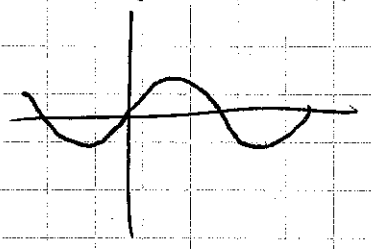
iv) $x = -\frac{\pi}{2}, 0$

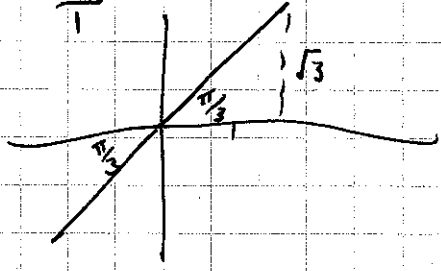
---

c) $\tan x \sin x - \sqrt{3}\sin x = 0$

$\sin x(\tan x - \sqrt{3}) = 0$

$\sin x = 0 \quad \tan x = \frac{\sqrt{3}}{1}$





i) $x = 0, \pi, \frac{\pi}{3}, \frac{4\pi}{3}$

ii) $x = 0, 180, 60°, 240°$

iii) $x = -\pi, 0, -\frac{2\pi}{3}, \frac{\pi}{3}$

iv) $x = \frac{\pi}{3}$

---

d) $\sqrt{3}\sec^2 x + 2\sec x = 0$

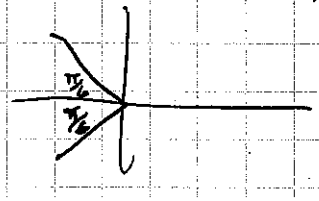
$\sec x(\sqrt{3}\sec x + 2) = 0$

$\sec x = 0 \quad \sec x = \frac{-2}{\sqrt{3}}$

$\frac{1}{\cos x} = 0 \quad \frac{1}{\cos x} = \frac{-2}{\sqrt{3}}$

$\cos x = \frac{1}{0} \quad \cos x = -\frac{\sqrt{3}}{2}$

Undefined



i) $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

ii) $x = 150°, 210°$

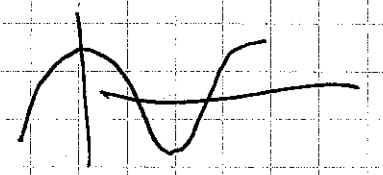
iii) $x = \frac{5\pi}{6}, -\frac{5\pi}{6}$

iv) $x = \{\}$

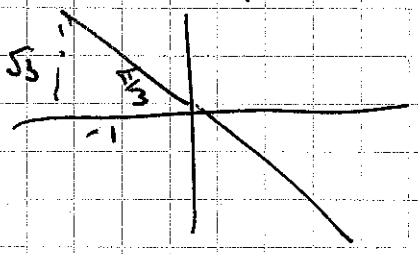
e) $\sqrt{3}\cos x \cot x + \cos x = 0$

$\cos x(\sqrt{3}\cot x + 1) = 0$

$\cos x = 0 \quad \cot x = -\frac{1}{\sqrt{3}}$



$\tan x = -\frac{\sqrt{3}}{1}$



i) $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{3}$

ii) $x = 90°, 270°, 120°, 300°$

iii) $x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{2\pi}{3}, -\frac{\pi}{3}$

iv) $x = -\frac{\pi}{2}, -\frac{\pi}{3}$

---

f) $4\cos x = 3\sec x$

$4\cos x - 3\sec x = 0$

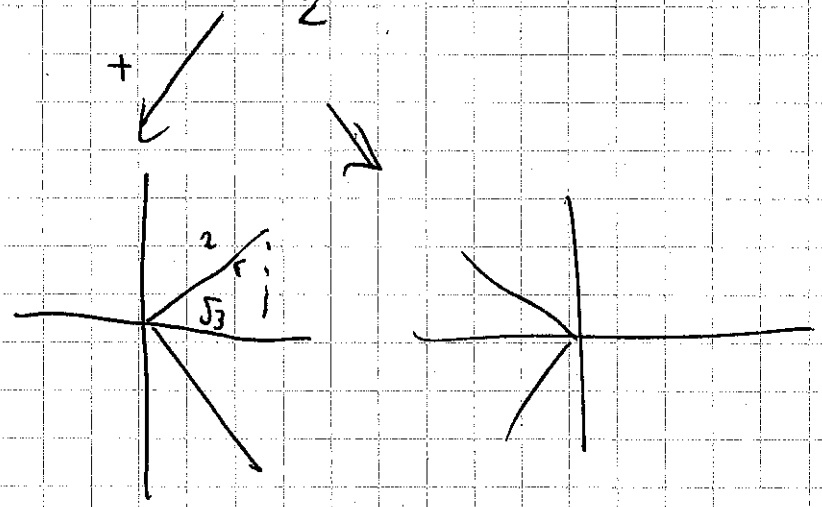
$4\cos x - 3\frac{1}{\cos x} = 0$

$4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\sqrt{\frac{3}{4}}$

$\cos x = \pm\frac{\sqrt{3}}{2}$



i) $x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$

ii) $x = 30°, 150°, 210°, 330°$

iii) $x = \frac{\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}, -\frac{5\pi}{6}$

iv) $x = \frac{\pi}{6}, -\frac{\pi}{6}$.

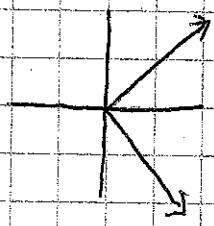
f) $4\cos x = 3\sec x$

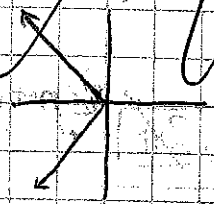
$4\cos x - 3\sec x = 0$

$4\cos x - 3\left(\frac{1}{\cos x}\right) = 0 \quad \cdot\cos x$

$4\cos^2 x - 3 = 0$

$\cos x = 0.8660$ or $\cos x = -0.8660$





$x_1 = 0.524$

$x_2 = 5.760$

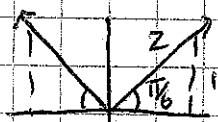
$x_3 = 2.618$

$x_4 = 3.665$

2. a) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = 1$



$x_1 = \frac{\pi}{6}$

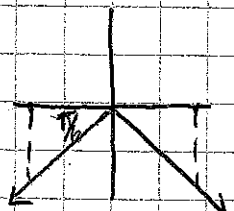
$x_2 = \frac{5\pi}{6}$

$x_3 = \frac{\pi}{2}$

b) $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$



$x_1 = \frac{7\pi}{6}$

$x_2 = \frac{11\pi}{6}$

$x_3 = \frac{\pi}{2}$

5.6 continued...
6.7

c) $\cos^2 x + 3\cos x + 2 = 0$

$(\cos x + 2)(\cos x + 1) = 0$

$\cos x = -2$ or $\cos x = -1$

↓

Does not exist

$x = \pi$

d) $2\cos^2 x - 3\cos x + 1 = 0$

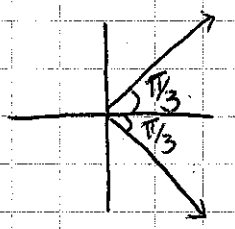
$(2\cos x - 1)(\cos x - 1)$

$\cos x = \frac{1}{2}$ $\cos x = 1$

$x_1 = \frac{\pi}{3}$

$x_2 = \frac{5\pi}{3}$

$x_3 = 0$



e) $2\tan^2 x = 3\tan x - 1$

$2\tan^2 x - 3\tan x + 1 = 0$

$(2\tan x - 1)(\tan x - 1) = 0$

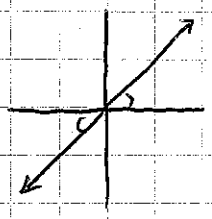
$\tan x = \frac{1}{2}$ or $\tan x = 1$

$x_1 = 0.46$

$x_2 = 3.61$

$x_3 = 0.79$

$x_4 = 3.93$



f) $3\tan^2 x = 2\tan x + 4$

$3\tan^2 x - 2\tan x - 4 = 0$

$\tan x = \frac{2 \pm \sqrt{52}}{6}$

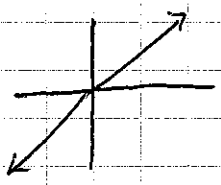
$\tan x = 1.5352$ $\tan x = -0.8685$

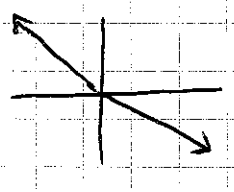
$x_1 = 0.99$

$x_2 = 4.14$

$x_3 = 2.43$

$x_4 = 5.57$



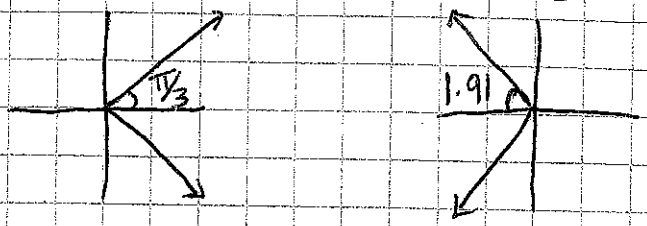


g) $\sec^2 x + \sec x - 6 = 0$

$(\sec x - 2)(\sec x + 3) = 0$

$\sec x = 2$ or $\sec x = -3$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -\frac{1}{3}$



$x_1 = \frac{\pi}{3}$

$x_2 = \frac{5\pi}{3}$

$x_3 = 1.23$

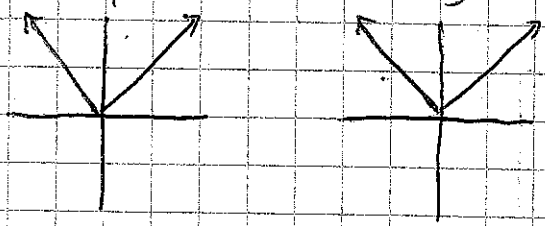
$x_4 = 5.05$

h) $\csc^2 x - 9\csc x + 20 = 0$

$(\csc x - 4)(\csc x - 5) = 0$

$\csc x = 4$ or $\csc x = 5$

$\sin x = \frac{1}{4}$ $\quad$ $\sin x = \frac{1}{5}$



$x_1 = 0.253$

$x_2 = 2.889$

$x_3 = 0.201$

$x_4 = 2.940$

3. a) $2\sin^2 x + \sin x = 0 \quad -\pi \le x < \pi$

$x = 0, \cancel{\pi}, \frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad$ $x = 0, -\frac{\pi}{6}, -\frac{5\pi}{6}, -\pi$

not in domain

b) $\cos^2 x - \cos x = 0 \quad -\pi \le x < \pi$

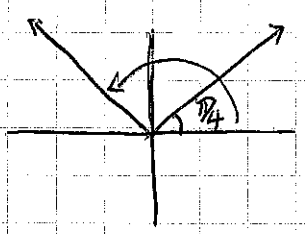
$x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $x = 0, \frac{\pi}{2}, -\frac{\pi}{2}$

c) $2\tan x \sin x - \sqrt{3}\tan x = 0 \quad -\pi \le x < \pi$

$x = -\pi, 0, \frac{\pi}{3}, \frac{2\pi}{3}$ $\quad$ $x = -\pi, 0, \frac{\pi}{3}, \frac{2\pi}{3}$

Section 6.8

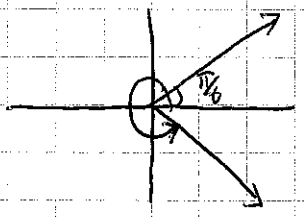
1. a) $\sin 2x = \frac{1}{\sqrt{2}}$



$2x_1 = \pi/4 \rightarrow \boxed{\pi/8}$

$2x_2 = \frac{3\pi}{4} \rightarrow \boxed{\frac{3\pi}{8}}$

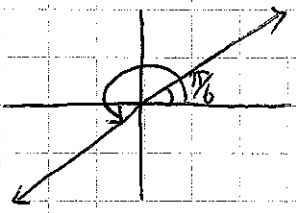
b) $\cos 4x = \frac{\sqrt{3}}{2}$



$4x_1 = \pi/6 \rightarrow \boxed{\pi/24}$

$4x_2 = \frac{11\pi}{6} \rightarrow \boxed{\frac{11\pi}{24}}$

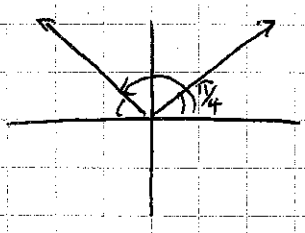
c) $\tan 2x = \frac{1}{\sqrt{3}}$



$2x_1 = \pi/6 \rightarrow \boxed{\pi/12}$

$2x_2 = 7\pi/6 \rightarrow \boxed{\frac{7\pi}{12}}$

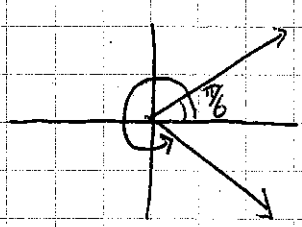
d) $\sin\left(\frac{1}{2}x\right) = \frac{1}{\sqrt{2}}$



$\frac{1}{2}x_1 = \pi/4 \rightarrow \boxed{\pi/2}$

$\frac{1}{2}x_2 = \frac{3\pi}{4} \rightarrow \boxed{\frac{3\pi}{2}}$

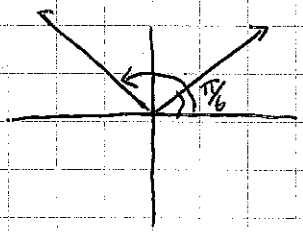
e) $\cos\left(\frac{1}{3}x\right) = \frac{\sqrt{3}}{2}$



$\frac{1}{3}x_1 = \pi/6 \rightarrow \boxed{\pi/2}$

$\frac{1}{3}x_2 = \frac{11\pi}{6} \rightarrow \boxed{\frac{11\pi}{2}}$

f) $\sin\left(\frac{1}{3}x\right) = \frac{1}{2}$



$\frac{1}{3}x_1 = \pi/6 \rightarrow \boxed{\pi/2}$

$\frac{1}{3}x_2 = \frac{5\pi}{6} \rightarrow \boxed{\frac{5\pi}{2}}$

Section 5.B continued...

2. a) $2\sin x\cos x + 1 = 0$

$\sin 2x + 1 = 0$

$\sin 2x = -1$

$2x = \frac{3\pi}{2}$ $\quad 2x = \frac{3\pi}{2} + 2\pi$

$x = \frac{3\pi}{4}$ $\quad 2x = \frac{7\pi}{2}$

$x = \frac{7\pi}{4}$

General Solution: $\boxed{x = \frac{3\pi}{4} + \pi n,\ n \in I}$

b) $\cos^2 x - \sin^2 x = 1$

$\cos 2x = 1$

$2x = 0$

$x = 0$

$2x = 0 + 2\pi$

$2x = 2\pi$

$x = \pi$

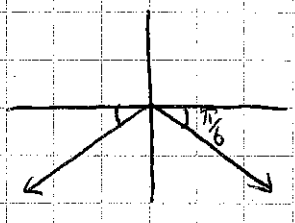
General Solution: $\boxed{x = \pi n,\ n \in I}$

c) $4\sin x\cos x + 1 = 0$

$2(2\sin x\cos x) + 1 = 0$

$2(\sin 2x) + 1 = 0$

$\sin 2x = -\frac{1}{2}$



$2x = 2\pi - \frac{\pi}{6}$

$2x = \frac{11\pi}{6}$ $\quad 2x = \frac{11\pi}{6} + 2\pi$

$x = \frac{11\pi}{12}$ $\quad = \frac{23\pi}{6}$

$x = \frac{23\pi}{12}$

$2x = \pi + \frac{\pi}{6}$

$2x = \frac{7\pi}{6}$

$x = \frac{7\pi}{12}$

$2x = \frac{7\pi}{6} + 2\pi$

$= \frac{19\pi}{6}$

$x = \frac{19\pi}{12}$

General Solution: $\boxed{x = \frac{7\pi}{12} + \pi n,\ n \in I}$ $\boxed{x = \frac{11\pi}{12} + \pi n,\ n \in I}$

d) $4\sin 2x\cos 2x + 1 = 0$

$2(2\sin 2x\cos 2x) + 1 = 0$

$2(\sin 4x) + 1 = 0$

$\sin 4x = -\frac{1}{2}$

$4x = \frac{7\pi}{6}$ $\quad 4x = \frac{19\pi}{6}$

$x = \frac{7\pi}{24}$ $\quad x = \frac{19\pi}{24}$

$4x = \frac{11\pi}{6}$

$x = \frac{11\pi}{24}$

$4x = \frac{23\pi}{6}$

$x = \frac{23\pi}{24}$

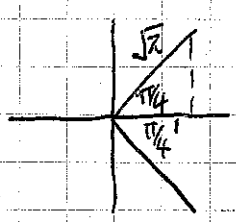
General Solution: $\boxed{x = \frac{7\pi}{12} + \frac{\pi}{2}n,\ n \in I}$ $\boxed{x = \frac{11\pi}{12} + \frac{\pi}{2}n,\ n \in I}$

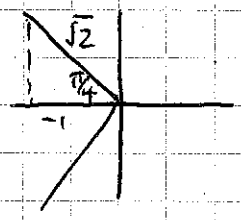
Section 5.8 continued...

e) $4\cos^2 x - 2 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \frac{1}{\sqrt{2}}$ or $\cos x = -\frac{1}{\sqrt{2}}$





$x_1 = \frac{\pi}{4}$ $x_2 = \frac{7\pi}{4}$ $x_3 = \frac{3\pi}{4}$ $x_4 = \frac{5\pi}{4}$

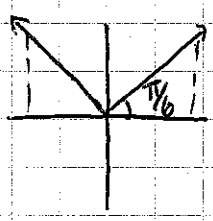
General solution: $x = \frac{\pi}{4} + 2\pi n$, $\frac{7\pi}{4} + 2\pi n$, $\frac{3\pi}{4} + 2\pi n$, $\frac{5\pi}{4} + 2\pi n$ $\}$ $n \in I$

f) $\sin(2x) - \cos x = 0$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\sin x = \frac{1}{2}$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

General solution: $x = \frac{\pi}{2} + 2\pi n$, $\frac{3\pi}{2} + 2\pi n$, $\frac{\pi}{6} + 2\pi n$, $\frac{5\pi}{6} + 2\pi n$ $\}$ $n \in I$

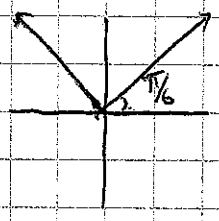
g) $\sin x = \cos 2x$

$0 = 1 - 2\sin^2 x - \sin x$

$(2\sin x - 1)(-\sin x - 1)$

$(-(+\sin x + 1))$

$\sin x = \frac{1}{2}$ $\quad \sin x = -1$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{3\pi}{2}$

General solution: $x = \begin{cases} \frac{\pi}{6} + 2\pi n \\ \frac{5\pi}{6} + 2\pi n \\ \frac{3\pi}{2} + 2\pi n \end{cases}, n \in I$

h) $\sin 2x - 2\cos^2 x = 0$

$2\sin x \cos x - 2\cos^2 x = 0$

$2\cos x(\sin x - \cos x) = 0$

$2\cos x = 0$ $\quad \sin x - \cos x = 0$

$\cos x = 0$ $\quad \sin x = \cos x$

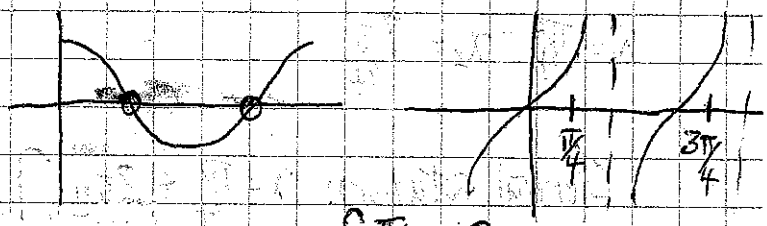
$\frac{\sin x}{\cos x} = 1 \quad \tan x = 1$

$x_1 = \frac{\pi}{2}$

$x_2 = \frac{3\pi}{2}$

$x_3 = \frac{\pi}{4}$

$x_4 = \frac{3\pi}{4}$



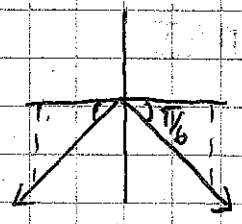
General Solution: $x = \begin{cases} \frac{\pi}{2} + 2\pi n \\ \frac{3\pi}{2} + 2\pi n \\ \frac{\pi}{4} + 2\pi n \\ \frac{3\pi}{4} + 2\pi n \end{cases}, n \in I$

3. $-\pi \leq x < \pi$ $\quad \cos 2x - 3\sin x = 2$

$1 - 2\sin^2 x - 3\sin x = 2$

$0 = (2\sin x + 1)(\sin x + 1)$

$\sin x = -\frac{1}{2}$ $\quad \sin x = -1$



$\boxed{x = -\frac{\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{2}}$

5.7 / 6.8 continued...

4. $\cos 4x = 0.6$

$x = 0.23$

$x = 1.34$

General solution: $x = \begin{cases} 0.23 + \frac{\pi}{2}n \\ 1.34 + \frac{\pi}{2}n \end{cases}, n \in I$

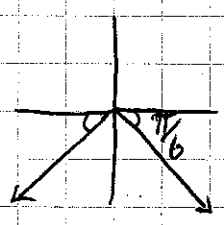
5. $-\pi \leq x < \pi \quad \cos(2x) - 3\sin x = 2$

$1 - 2\sin^2 x - 3\sin x = 2$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2} \quad \sin x = -1$



$x = -\frac{\pi}{6}, -\frac{5\pi}{6}$

$x = -\frac{\pi}{2}$

Section 5.8 6.9

1. a) $\sin 2x - \cos x = 1$ $x = 4.36, 3.14$

b) $3\sin x = x + 1$ $x = 0.54, 1.87$

c) $\tan x - \cos x = -2$ $x = 1.97, 5.32$

d) $\sin x = \frac{1}{x}$ $x = 1.05, 2.77$

e) $3\cos 2x = -x$ $x = 0.95, 1.99$

f) $3\tan\left(\frac{1}{2}x - 2\right) = 4\sin 2x$ $x = 2.20, 2.90, 4.60$

g) $2\cos x = 2^x$ $x = 0.66$

h) $4\sin x = \log x$ $x = 3.02$