CHAIN RULE PART II

Note Title 3/3/2014

RECALL: $f(x) = (x^2+6)^3(x^4+3x)^2$

$$f'(x) = \underbrace{3(x^2+6)^2(2x)}(x^4+3x)^2 + \underbrace{2(x^4+3x)(4x^3+3)}(x^2+6)^3$$

$$= 6x(x^2+6)^2(x^4+3x)^2 + (8x^3+6)(x^4+3x)(x^2+6)^3$$

$$= (x^2+6)^2(x^4+3x)\left[6x(x^4+3x) + (8x^3+6)(x^2+6)\right]$$

EX $f(x) = \dfrac{(2x+3)^2}{(x+1)^3}$ FIND $f'(x)$

$$f'(x) = \frac{2(2x+3)(2)(x+1)^3 - 3(x+1)^2(1)(2x+3)^2}{(x+1)^6}$$

$$= \frac{4(2x+3)(x+1)^3 - 3(x+1)^2(2x+3)^2}{(x+1)^{3\&4}}$$

$$= \frac{(2x+3)[4(x+1) - 3(2x+3)]}{(x+1)^4}$$

$$= \frac{(2x+3)[4x+4 - 6x - 9]}{(x+1)^4}$$

$$= \frac{(2x+3)(-2x-5)}{(x+1)^4}$$

THE DERIVATIVE OF COMPOSITION OF FUNCTIONS

GIVEN $f(x) = 2x+3$ AND $g(x) = x^2+4$

FIND $f(g(x)) = f(x^2+4) = 2(x^2+4)+3$

$= 2x^2+8+3$

$= 2x^2+11$

$f \circ g(x)$

FIND $g \circ f(x) = g(f(x))$

$= g(2x+3) = (2x+3)^2+4$

$= 4x^2+12x+9+4$

$= 4x^2+12x+13$

$$\frac{d}{dx} f \circ g(x) = \frac{d}{dx} f(g(x))$$

$$= f'(g(x)) \cdot g'(x)$$

Ex $g(x) = 4x^9 + 5x^3 + 6x$

$f(x) = \sqrt{x}$ Find $\frac{d}{dx} f(g(x))$

$$f(g(x)) = \sqrt{4x^9 + 5x^3 + 6x}$$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$ or $\frac{1}{2\sqrt{x}}$

$g'(x) = 36x^8 + 15x^2 + 6$

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$= \frac{1}{2\sqrt{4x^9+5x^3+6x}} \cdot 36x^8+15x^2+6$$

$$= \frac{36x^8+15x^2+6}{2\sqrt{4x^9+5x^3+6x}}$$

— THE OTHER FORM OF THE CHAIN RULE

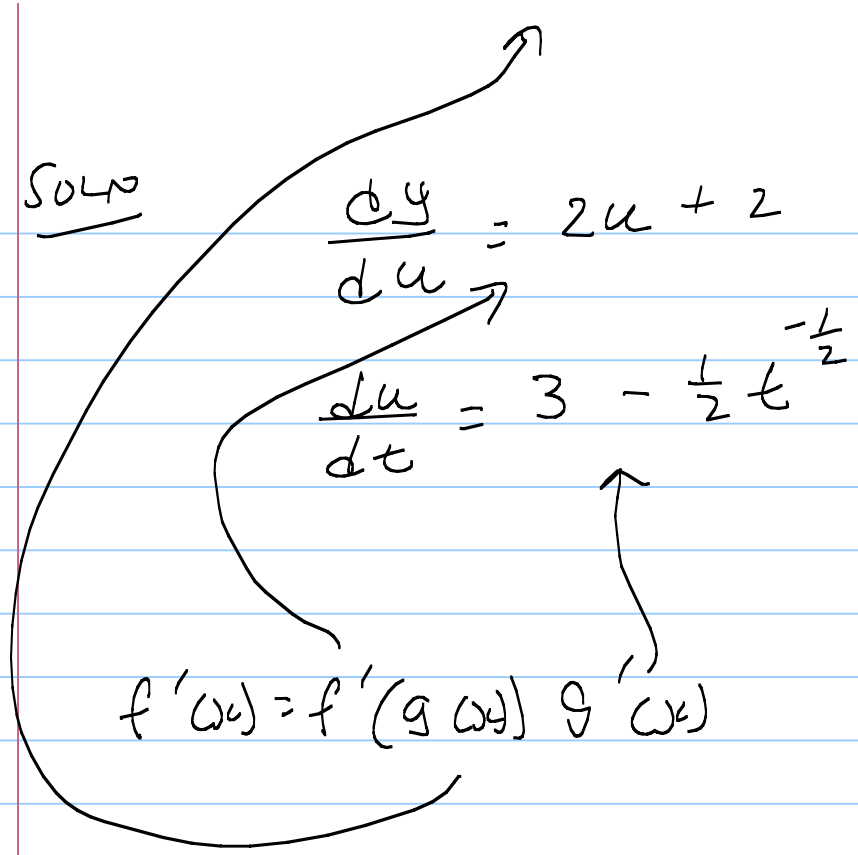
SUPPOSE $y = f(u)$ AND $u = g(t)$

$\therefore\ y = f(g(t))$

THEN $\dfrac{dy}{dt} = \dfrac{dy}{du} \cdot \dfrac{du}{dt}$

EG $y = u^2 + 2u$

$u = 3t - \sqrt{t}$



$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$$

$$\frac{dy}{dt} = (2u + 2) \cdot \left(3 - \frac{1}{2}t^{-\frac{1}{2}}\right)$$

$$\frac{dy}{dt} = \left(2(3t - \sqrt{t}) + 2\right)\left(3 - \frac{1}{2}t^{-\frac{1}{2}}\right)$$

If $y = 4x^3 + 5x^9 + 7$

$x = 3t^4 + 8$

Find $\frac{dy}{dt}$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\frac{dy}{dx} = 12x^2 + 45x^8$$

$$\frac{dx}{dt} = 12t^3$$

$$\frac{dy}{dt} = (12x^2 + 45x^8)(12t^3)$$

$$\frac{dy}{dt} = (12(3t^4+8)^2 + 45(3t^4+8)^8)(12t^3)$$

ADD

Pg 160 # 41-46, 51, 52, 68