BINOMIAL THEOREM

Note Title 1/17/2013

IE EXPAND (1) $(a+b)^6$ AND (2) $(2x+3)^4$

SOLN THE NUMERICAL COEFFICIENTS IN THE EXPANSION CAN BE FOUND FROM PASCAL'S TRIANGLE. THE ROW WHOSE SECOND ENTRY MATCHES THE EXPONENT IS THE CORRECT ROW.

7^{TH} ROW 1 6 15 20 15 6 1

$\therefore$ THE EXPANSION IS OF THE FORM

$(a+b)^6 = 1(\ )(\ ) + 6(\ )(\ ) + 15(\ )(\ ) + 20(\ )(\ ) + 15(\ )(\ )$
$+ 6(\ )(\ ) + 1(\ )(\ )$

- THE TERMS ARE MADE UP OF DECREASING POWERS OF "a" FROM 6 TO 0 AND INCREASING POWERS OF "b" FROM 0 TO 6

$(a+b)^6 = 1(a^6)(b^0) + 6(a^5)(b^1) + 15(a^4)(b^2) + 20(a^3)(b^3) + 15(a^2)(b^4)$
$+ 6(a^1)(b^5) + 1(a^0)(b^6)$
$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

(2) NUMERICAL COEFFICIENT OCCURS IN THE 5^{TH} ROW

1 4 6 4 1

$$(2x+3)^4 = \cancel{1}(2x)^4\cancel{(3)^0} + 4(2x)^3(3)^1 + 6(2x)^2(3)^2 + 4(2x)^1(3)^3 + 1(2x)^0(3)^4$$

$$= 16x^4 + 96x^3 + 216x^2 + 216x + 81$$

* RECALL: EACH ENTRY OF PASCAL'S TRIANGLE CAN BE WRITTEN IN THE FORM ${}_nC_r$ *

THE BINOMIAL THEOREM – THE EXPANSION OF $(a+b)^n$, WHERE n IS A NATURAL NUMBER IS GIVEN BY

$$(a+b)^n = {}_nC_0\, a^n b^0 + {}_nC_1\, a^{n-1} b^1 + {}_nC_2\, a^{n-2} b^2 + \ldots {}_nC_n\, a^0 b^n$$

THE GENERAL TERM IS OF THE FORM

$$t_{r+1} = {}_nC_r\, a^{n-r} b^r$$

WHERE $r=0$ GIVES THE 1^{ST} TERM, $r=1$ GIVES THE SECOND AND SO ON.

<u>IE</u> (1) USE BINOMIAL THEOREM TO EXPAND $(x+3)^5$

(2) DETERMINE THE GENERAL TERM AND THE 5^{TH} TERM IN THE EXPANSION OF $(2x+5)^7$

<u>SOLN</u> (1) $a = x$, $b = 3$, $n = 5$

$$= {}_5C_0\, x^{5-0}\, 3^0 + {}_5C_1\, x^{5-1}\, 3^1 + {}_5C_2\, x^{5-2}\, 3^2 + {}_5C_3\, x^{5-3}\, 3^3$$

$$+ {}_5C_4\, x^{5-4}\, 3^4 + {}_5C_5\, x^{5-5}\, 3^5$$

$$= x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243$$

(2) FOR GENERAL TERM USE $a = 2x$ $b = 5$ $n = 7$

$$t_{r+1} = {}_nC_r \, a^{n-r} b^r$$

$$t_{r+1} = {}_7C_r \, (2x)^{7-r} (5)^r$$

* FOR THE 5^{TH} TERM r IS ONE LESS SO $r = 4$

$$t_{4+1} = {}_7C_4 \, (2x)^{7-4} (5)^4$$

$$t_5 = 35(8x^3)(625)$$

$$= 175000x^3$$

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