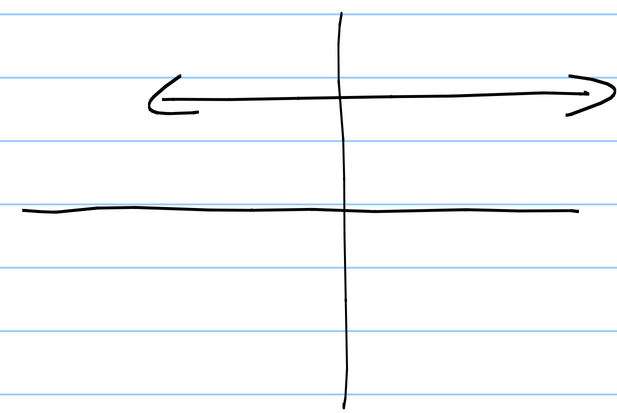
ALGEBRAIC LIMITS

Note Title 2/11/2009

NOTES ABOUT CONTINUITY

(1) ANY CONSTANT FUNCTION IS CONTINUOUS

IE $f(x) = c$



(2) IF $f(x)$ AND $g(x)$ ARE CONTINUOUS THEN

$f(x) + g(x)$ AND $f(x) - g(x)$ AND $f(x) \cdot g(x)$

ARE CONTINUOUS ALSO

3. IF $f(x)$ AND $g(x)$ ARE CONTINUOUS THEN

$\frac{f(x)}{g(x)}$ IS CONTINUOUS WHERE $g(x) \neq 0$

- IF A FUNCTION f IS CONTINUOUS AT a THEN WE ARE ABLE TO SUBSTITUTE TO FIND THE LIMIT.

IE $$\lim_{x \to 4} x^2 - 2x - 10 = (4)^2 - 2(4) - 10$$
$$= -2$$

IE $$\lim_{x \to 10} \sqrt{2x - 4} = \sqrt{2(10) - 4} = \sqrt{16} = 4$$

IE $\lim_{x \to -3} \sqrt{2x-4} = \sqrt{2(-3)-4} = \sqrt{-10}$

D.N.E $\because$ NOT CONTINUOUS AT -3

IE $\lim_{x \to 2} \dfrac{x^2-7x+10}{x^2-5x+6} = \lim_{x\to 2} \dfrac{(x-5)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)}$

$= \lim_{x \to 2} \dfrac{x-5}{x-3}$

$= \dfrac{2-5}{2-3} = \dfrac{-3}{-1} = 3$

IE $\lim_{x \to 0} 2x^2 + 3x^3h + 5x + 3h + 9$

$= \cancel{2(0)^2} + \cancel{3(0)^3h} + \cancel{5(0)} + 3h + 9$

$= 3h + 9$

LIMIT RULES

GIVEN $\boxed{\lim_{x \to a} f(x) = L}$, $\boxed{\lim_{x \to a} g(x) = M}$

1.) $\lim_{x \to a} c = c$ IE $\lim_{x \to 20} 5 = 5$

2.) $\lim_{x \to a} (f(x))^n = \left(\lim_{x \to a} f(x)\right)^n = L^n$

3.) $\lim\limits_{x \to a} \sqrt[n]{f(x)} = \lim\limits_{x \to a} \left(f(x) \right)^{\frac{1}{n}} = L^{\frac{1}{n}}$, $f(x) \geq 0$ AS $x \to a$

4.) $\lim\limits_{x \to a} \left(f(x) \pm g(x) \right) = \lim\limits_{x \to a} f(x) \pm \lim\limits_{x \to a} g(x) = L \pm M$

5.) $\lim\limits_{x \to a} \left(f(x) \cdot g(x) \right) = L \cdot M$

6.) $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}$, IF $g(x) \neq 0$ AS $x \to a$

7.) $\lim\limits_{x \to a} c\, f(x) = c \lim\limits_{x \to a} f(x) = c \cdot L$

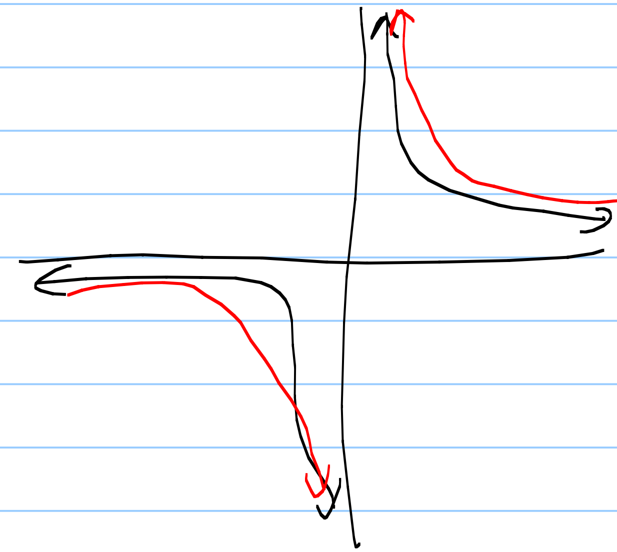
LIMITS AND INFINITY

$$\lim_{x \to 0} \frac{1}{x} = \text{D.N.E.}$$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

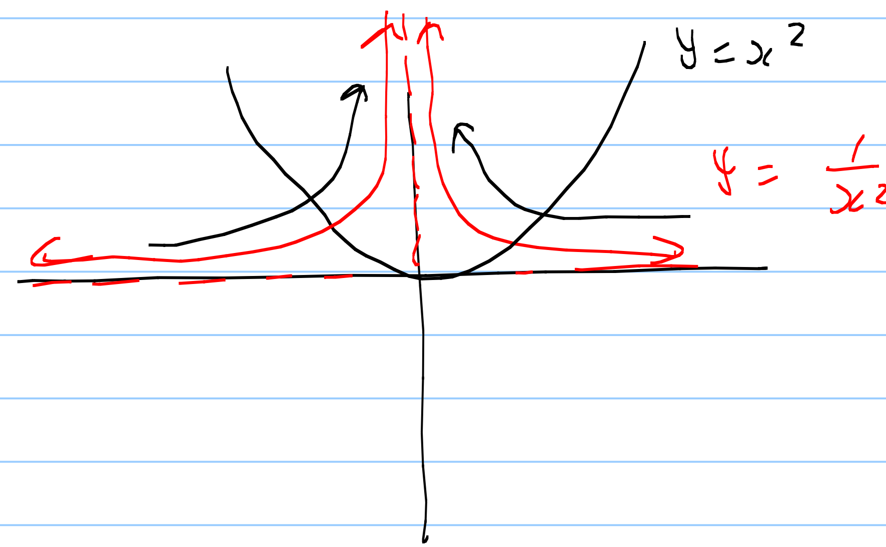
NOT ACTUAL #'S



SO

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$

$$\lim_{x \to 0^+} \frac{1}{x^2} = \infty$$



$$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$$

LIMITS INVOLVING INFINITY

$$\boxed{\lim_{x \to \infty} \frac{1}{x} = 0}$$

$$\lim_{x \to \infty} \frac{3x^2 + 5x + 2}{2x^2 + 2x + 1} = \lim_{x \to \infty} \frac{\frac{3x^2}{x^2} + \frac{5x}{x^2} + \frac{2}{x^2}}{\frac{2x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2}}$$

$$= \lim_{x \to \infty} \frac{3 + \frac{5}{x} + \frac{2}{x^2}}{2 + \frac{2}{x} + \frac{1}{x^2}}$$

$$= \frac{3 + 0 + 0}{2 + 0 + 0} = \frac{3}{2}$$

IE $\lim\limits_{x \to \infty} \dfrac{3x^2+4x+5}{3x^4+2x+9} = \lim\limits_{x \to \infty} \dfrac{\frac{3x^2}{x^4} + \frac{4x}{x^4} + \frac{5}{x^4}}{\frac{3x^4}{x^4} + \frac{2x}{x^4} + \frac{9}{x^4}}$

$= \lim\limits_{x \to \infty} \dfrac{\frac{3}{x^2} + \frac{4}{x^3} + \frac{5}{x^4}}{3 + \frac{2}{x^3} + \frac{9}{x^4}} = \dfrac{0+0+0}{3+0+0} = \dfrac{0}{3} = 0$

IE $\lim\limits_{x \to \infty} \dfrac{x^2+2x}{x+1} = \lim\limits_{x \to \infty} \dfrac{\frac{x^2}{x} + \frac{2x}{x}}{\frac{x}{x} + \frac{1}{x}}$

$= \lim\limits_{x \to \infty} \dfrac{x+2}{1+\frac{1}{x}} = \infty$

$\Rightarrow \dfrac{\infty+2}{1+0} = \dfrac{\infty+2}{1}$

H/w Pg 100 #1–16 ALL
#23, 24
#31 – 45 ODD