

## 4.4 DECAY

- IN A POPULATION MODEL,  $k$  IS GIVEN BY BIRTHRATE - DEATHRATE. IF  $k < 0$  THEN THE POPULATION IS DECREASING (DECREASES).

$$\frac{dP}{dt} = -kP \quad \therefore P = ce^{-kt}$$

THE OXYGEN-14 DECAYS AT A RATE OF 15% / Hour  
∴ THE RATE OF CHANGE OF THE AMOUNT  $N$  IS

$$\text{GIVEN BY } \frac{dN}{dt} = -0.15N$$

A) FIND EQN

B) SUPPOSE  $N = 50$  AT TIME  $t = 0$ , HOW MUCH REMAINS AFTER 1 DAY?

C) FIND THE HALF LIFE OF 0-14

SOLN

A)  $N = Ce^{-0.15t}$

B)  $50 = Ce^{-0.15(0)}$

$$50 = C$$

$$N = 50e^{-0.15t}$$

$$N = 50e^{-0.15(24)}$$

$$N = 1.37$$

$$c) \quad 25 = 50 e^{-.15t}$$

$$\frac{1}{2} = e^{-.15t}$$

$$\ln \frac{1}{2} = \frac{-.15t}{-.15}$$

$$4.62 \text{ hrs.} = t$$

### NEWTON'S LAW OF COOLING

- THE TEMPERATURE  $T$  OF A COOLING OBJECT

DROPS AT A RATE THAT IS PROPORTIONAL TO

THE DIFFERENCE  $T - C$ , WHERE  $C$  IS THE

TEMPERATURE OF THE MEDIUM (SURROUNDING AREA).

$$\therefore \frac{dT}{dt} = -k(T - C)$$

THE EQUATION IS  $T = Ae^{-kt} + C$

IE A METAL PLATE IS HEATED TO  $500^{\circ}\text{C}$

AND IS PLACED IN A COOLING CHAMBER @  $20^{\circ}\text{C}$ ,

THE PLATE COOLS TO  $450^{\circ}\text{C}$  IN 15 MINS.

A) WHAT IS THE TEMPERATURE AFTER 2 HRS?

B) HOW LONG DOES IT TAKE TO COOL TO  $100^{\circ}\text{C}$ ?

~~S&B~~

$$A) T = a e^{-kt} + c$$

$$560 = a e^{-1k(10)} + 20$$

$$560 = a + 20$$

$$480 = a$$

$$T = 480 e^{-kt} + 20$$

$$450 = 480 e^{-1k(15)} + 20$$

$$\frac{430}{480} = \frac{480 e^{-15k}}{480}$$

$$0,8958 = e^{-15k}$$

$$\ln \frac{0.8958}{-15} = \frac{-15k}{-15}$$

$$.0073 = k$$

$$T = 480 e^{-.0073t} + 20$$

$$T = 480 e^{-.0073(120)} + 20$$

$$T = 219.89^{\circ}$$

$$B) \quad 100 = 480 e^{-.0073t} + 20$$

$$\frac{80}{480} = \frac{480 e^{-.0073t}}{480}$$

$$\frac{1}{6} = e^{-.0073t}$$

$$\ln \frac{1}{6} = -.0073t$$

245.45 mins = t

THE GUNNET ARRIVES AT THE SCENE OF A MURDER  
TO FIND MR. MORGAN DEAD, THE TIME IS  
3:30 PM. SHE FINDS THE BODY TEMPERATURE TO  
BE  $28.56^{\circ}\text{C}$ . SHE WAITS AN HOUR AND FINDS  
THE BODY TEMPERATURE TO BE  $25.94^{\circ}\text{C}$ . THE  
TEMPERATURE OF THE AIR IS  $15.51^{\circ}\text{C}$ . THE  
FOLLOWING PEOPLE WERE SEEN AROUND THE AREA  
DURING THE DAY OF THE MURDER.

A) WILL 11:00-12:00 B) NIRAV 12:00-1:00 C) KATERINA 1:00-2:00

D) PHEBE 2:00 - 3:00

WTO KILLEN MR. MORRAN?

Soln  $T = a e^{-kt} + C$

$$37 = a e^{-k(0)} + 15.56$$

$$21.44 = a$$

$$T = 21.44 e^{-kt} + 15.56$$

$$28.56 = 21.44 e^{-kt} + 15.56$$

$$0.6063 = e^{-kt}$$

$$\ln \frac{0.6063}{-1} = \frac{-kt}{-1}$$

$$0.5603 = kt$$



$$25.94 = 21.44 e^{-k(t+1)} + 15.56$$

$$\frac{-15.56}{21.44} = \frac{-15.56}{21.44} e^{-k(t+1)}$$

$$0.4841 = e^{-k(t+1)}$$

$$\ln \cdot 4841 = \frac{-k(t+1)}{-1}$$

$$.7254 = k(t+1)$$

$$.7254 = k t + k$$

$$.7254 = 0.5003 + k$$

$$0.2251 = k$$

$$28.56 = 21.44 e^{-.2251t} + 15.56$$

2.22 = t

2.22 HOURS FROM TIME OF DEATH  
∴ KATERSINA DID IT !!

H/w Pg. 317 # 1, 9, 15, 20, 23, 25, 27, 35