USING DERIVATIVES TO FIND MAX + MIN VALUES

Note Title 3/26/2014

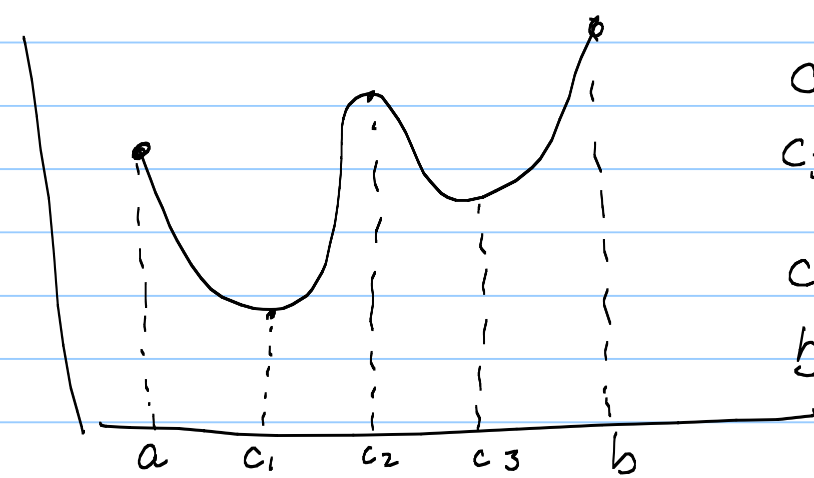
- MAX/MIN VALUES ON CLOSED INTERVALS $[a, b]$

ABSOLUTE/RELATIVE MAXIMUM/MINIMUM WILL OCCUR

AT EITHER CRITICAL POINTS INSIDE THE DOMAIN

OR AT THE END POINTS OF THE DOMAIN

EX



c_1 = ABSOLUTE MIN

c_3 = RELATIVE MIN

c_2 = RELATIVE MAX

b = ABSOLUTE MAX

THE EXTREME VALUE THEOREM

- A CONTINUOUS FUNCTION "f" DEFINED ON A CLOSED INTERVAL $[a, b]$ MUST HAVE AN ABSOLUTE MAXIMUM VALUE AND AN ABSOLUTE MINIMUM VALUE AT POINTS IN $[a, b]$

HOW TO FIND MAX/MIN

1) FIND $f'(x)$
2) FIND CP'S IN $[a, b]$
3) FIND THE Y-VALUES FOR CP'S IN THE

INTERVAL $[a, b]$ AS WELL AS THE Y-VALUES FOR THE END POINTS OF THAT INTERVAL

4) DETERMINE WHICH POINTS ARE ABSOLUTE/RELATIVE MAX/MIN

IE $f(x) = x^3 - 3x^2 \quad \underbrace{[0, 5]}$

SOLN $f'(x) = 3x^2 - 6x$

$$3x^2 - 6x = 0$$
$$3x(x - 2) = 0$$
$$x = 0 \qquad x = 2$$

$f(0) = 0$

$f(2) = -4 \leftarrow$ ABSOLUTE MIN $(2, -4)$

$f(5) = 50 \leftarrow$ ABSOLUTE MAX $(5, 50)$

- WHAT ABOUT FINDING ABSOLUTE/RELATIVE MAX/MIN ON OPEN INTERVALS? $\because$ THE INTERVAL IS OPEN, THERE ARE NO END POINTS TO TEST $\therefore$ WE NEED A DIFFERENT METHOD. WE CAN USE THE 2ND DERIVATIVE TO DETERMINE CONCAVITY AND SUBSEQUENTLY DETERMINE MAX/MIN

$$f''(x) > 0 \quad \text{CC UP} \Longrightarrow \text{MIN}$$

$$f''(x) < 0 \quad \text{CC DN} \Longrightarrow \text{MAX}$$

IE $f(x) = x^4 - 2x^2 \quad (-\infty, \infty)$

SOLN $f'(x) = 4x^3 - 4x$

$4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$x = 0 \quad x = -1 \quad x = 1$

$f(-1) = -1 \qquad f(0) = 0 \qquad f(1) = -1$

$\underbrace{(-1, -1)} \qquad (0, 0) \qquad \underbrace{(1, -1)}$

$f''(x) = 12x^2 - 4$

$f''(-1) = 8 \Leftarrow$ CC UP ABSOLUTE MIN

$f''(0) = -4 \Leftarrow$ CC DN RELATIVE MAX

$f''(1) = 8$ $\Leftarrow$ cc up ABSOLUTE MIN