3.3 ASYMPTOTES

REVIEW: CP'S $f'(x) = 0$ OR $f'(x) = DNE$

IP'S $f''(x) = 0$

$f''(x) > 0$ CC UP $f'(x) > 0$ INCREASING

$f''(x) < 0$ CC DOWN $f'(x) < 0$ DECREASING

ASYMPTOTES: - HORIZONTAL

- VERTICAL
- OBLIQUE / SLANT

IE $f(x) = \dfrac{x^2-4}{x^2-9}$, SKETCH + FIND ASYMPTOTES

VERTICAL ASYMPTOTES

$x = -3$ AND $x = 3$

$$\lim_{x \to 3} \frac{x^2-4}{x^2-9} = \text{DNE}$$

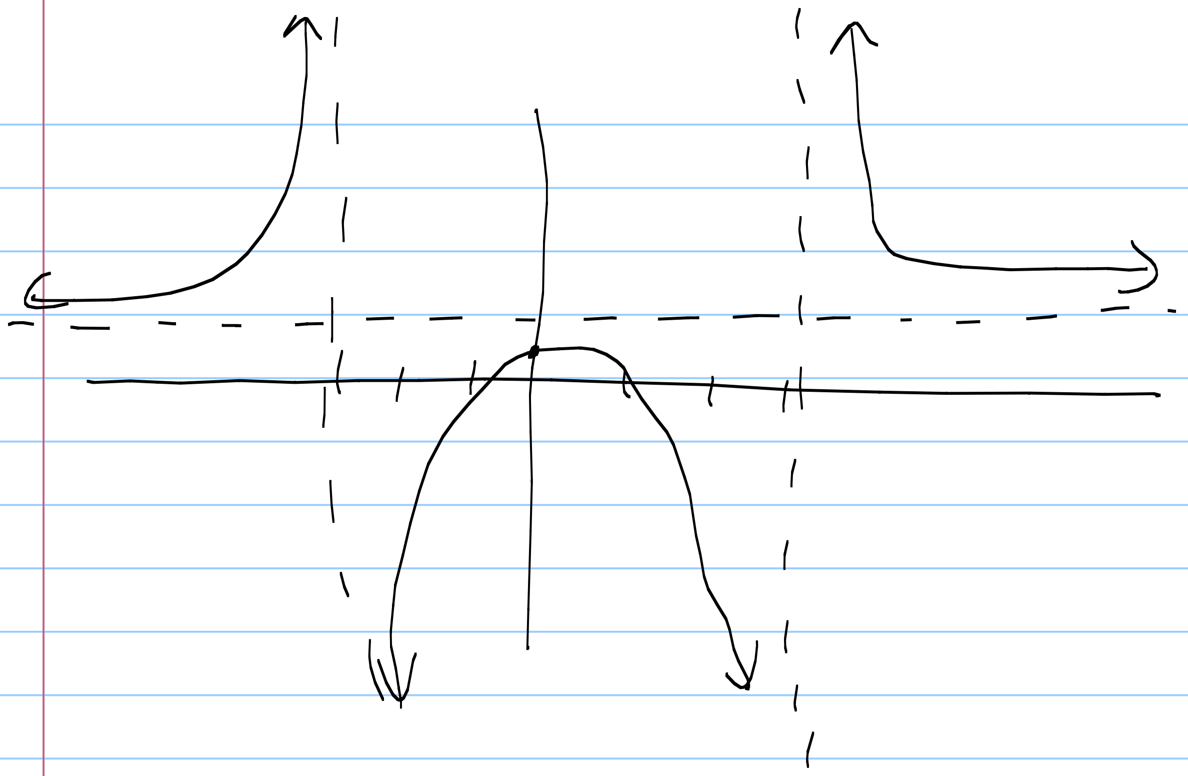
$$\lim_{x \to -3} \frac{x^2-4}{x^2-9} = \text{DNE}$$

HORIZONTAL ASYMPTOTES

$$\lim_{x \to \infty} \frac{x^2-4}{x^2-9}$$

$$\lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}}$$

$$\lim_{x \to \infty} \frac{1 - \frac{4}{x^2}}{1 - \frac{9}{x^2}}$$



$$\frac{1-0}{1-0} = 1 \therefore y = 1$$

FIND HORIZONTAL + VERTICAL ASYMPTOTES OF

$$y = \frac{2x+4}{x^2-6x-27}$$

VERTICAL ASYMPTOTES (DENOMINATOR = 0)

$$x^2 - 6x - 27 = 0$$

$$(x-9)(x+3) = 0$$

$$x = 9 \qquad x = -3$$

HORIZONTAL ASYMPTOTES

$$\lim_{x \to \infty} \frac{2x+4}{x^2 - 6x - 27}$$

$$\lim_{x \to \infty} \frac{\frac{2x}{x^2} + \frac{4}{x^2}}{\frac{x^2}{x^2} - \frac{6x}{x^2} - \frac{27}{x^2}}$$

$$\lim_{x \to \infty} \frac{\frac{2}{x} + \frac{4}{x^2}}{1 - \frac{6}{x} - \frac{27}{x^2}}$$

$$\frac{0+0}{1-0-0} = 0$$

$$y = 0$$

OBLIQUE ASYMPTOTES

THE LINE $y = mx + b$ IS AN OBLIQUE ASYMPTOTE OF THE RATIONAL FUNCTION $f(x) = \dfrac{P(x)}{Q(x)}$ IF $f(x)$ CAN BE WRITTEN AS $f(x) = (mx+b) + g(x)$ WHERE $g(x) \to 0$ AS $|x| \to \infty$. IN OTHER WORDS THE DEGREE OF THE NUMERATOR MUST BE GREATER THAN THE DEGREE OF THE DENOMINATOR FOR THIS TO OCCUR

SE $f(x) = \dfrac{6x^2 + 8x - 6}{3x - 2}$

VERT ASYMPTOTE

$3x - 2 = 0$

$3x = 2$

$x = \dfrac{2}{3}$

HORZ. ASYMPTOTE

$$\lim_{x \to \infty} \frac{6x^2 + 8x - 6}{3x - 2}$$

$$\lim_{x \to \infty} \frac{\frac{6x^2}{x} + \frac{8x}{x} - \frac{6}{x}}{\frac{3x}{x} - \frac{2}{x}}$$

$$\lim_{x \to \infty} \frac{6x + 8 - \frac{6}{x}}{3 - \frac{2}{x}}$$

$$= \frac{6(\infty) + 8 - 0}{3 - 0}$$

$= \infty$

$$f(x) = \frac{6x^2 + 8x - 6}{3x - 2}$$

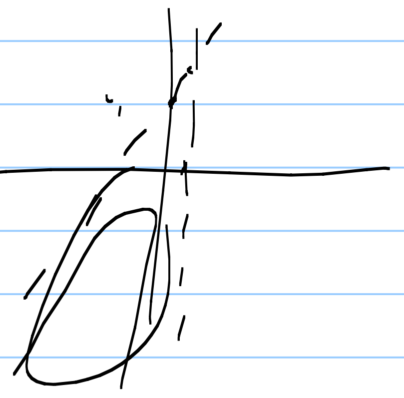
$$\begin{array}{r l}
 & 2x + 4 \\
3x - 2 \,) & 6x^2 + 8x - 6 \\
- & 6x^2 - 4x \quad \times \\
\hline
 & 12x - 6 \\
- & 12x - 8 \\
\hline
 & 2
\end{array}$$

✓

$$f(x) = 2x + 4 + \frac{2}{3x - 2}$$

$\therefore$ OBLIQUE ASYMPTOTE

✓ AT $y = 2x + 4$



GRAPHING : THE "FULL MEAL DEAL"

1.) INTERCEPTS (x & y) IF POSSIBLE

2.) ASYMPTOTES (HORZ, VERT, OBLIQUE)

3.) CRITICAL POINTS
    3A) INC / DEC TABLE
4.) INFLECTION POINTS
    4A) CC UP / CC DN TABLE

SKETCH $f(x) = \frac{x^2 - 9}{x + 1}$

<u>SOLN</u> x - INTERCEPT (TOP = 0)

$x^2 - 9 = 0$

$x = 3 \quad x = -3$

y - INTERCEPT ($x = 0$)

$\frac{0^2 - 9}{0 + 1} = -9$

ASYMPTOTES

VERTICAL (BOTTOM = 0)

$x + 1 = 0$

$x = -1$

HORIZONTAL

NONE

OBLIQUE

$$\begin{array}{r} x - 1 \\ x+1 \overline{)\, x^2 + 0x - 9} \\ \underline{x^2 + x } \\ -x - 9 \\ \underline{-x - 1} \\ -8 \end{array}$$

$y = x - 1$

$$f(x) = \frac{x^2 - 9}{x + 1}$$

CP's $\quad f'(x) = \dfrac{(2x)(x+1) - (x^2 - 9)}{(x+1)^2}$

$$= \frac{2x^2 + 2x - x^2 + 9}{(x+1)^2}$$

$$= \frac{x^2 + 2x + 9}{(x+1)^2}$$

UNDEFINED WHEN

$(x+1)^2 = 0$

$x = -1$

$x^2 + 2x + 9 = 0$

NO CP'S

	$-\infty \to -1$	-1	$-1 \to \infty$
$f'(x)$	$+$	∞	$+$
$f(x)$	INC	"CP"	INC

IP'S $\quad f'(x) = \dfrac{x^2 + 2x + 5}{(x+1)^2}$ $\qquad \dfrac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$

$$f''(x) = \frac{(2x+2)(x+1)^2 - (2)(x+1)(1)(x^2+2x+5)}{(x+1)^4}$$

$$f''(x) = \frac{(2x+2)(x+1)^{\cancel{2}} - 2(x+1)(x^2+2x+5)}{(x+1)^{\cancel{4}3}}$$

$$f''(x) = \frac{(2x+2)(x+1) - 2(x^2+2x+9)}{(x+1)^3}$$

$$f''(x) = \frac{\cancel{2x^2} \cancel{+4x} + 2 \cancel{-2x^2} \cancel{-4x} - 18}{(x+1)^3}$$

$$f''(x) = \frac{-16}{(x+1)^3}$$

$$f''(x) = 0 \qquad \frac{-16}{(x+1)^3} = 0 \qquad \text{NONE}$$

$$f''(x) = \text{DNE}$$

UNDEFINED @ $x = -1$

	$-\infty \to -1$	-1	$-1 \to \infty$
$f''(x)$	$+$	∞	$-$
$f(x)$	CC UP	IP	CC DN